UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
SCHOOL OF EDUCATIONAL DEVELOPMENT AND OUTREACH
INSTITUTE OF EDUCATION

FIVE-SEMESTER BACHELOR OF EDUCATION (SANDWICH) PROGRAMME
SECOND YEAR, END-OF-FIRST SEMESTER EXAMINATION, APRIL 2021

APRIL 28, 2021   VECTOR ALGEBRA AND DIFFERENTIAL   2:00 PM - 2:40 PM
                                        EQUATIONS

This paper consists of two sections, A and B. Answer ALL the questions in Section A and TWO questions from Section B. Section A will be collected after the first 40 minutes.

SECTION A

Answer ALL the questions in this Section.

Items 1 to 20 are stems followed by four options lettered A to D. Read each item carefully and circle the letter of the correct or best option.

1. Calculate the direction angle sustained by vector $A = 2i + j + 5k$ with respect to y-axis in the xyz coordinate system.
    A. $25.00°$
    B. $60.44°$
    C. $79.48°$
    D. $16.12°$

2. Which of the following vectors are collinear?
    A. $\langle 2,1,3 \rangle$ and $\langle -2,3,1 \rangle$
    B. $\langle 4,1,9 \rangle$ and $\langle 9,4,-1 \rangle$
    C. $\langle 2,-3,4 \rangle$ and $\langle -4,6,-8 \rangle$
    D. $\langle 1,-1,5 \rangle$ and $\langle 1,1,5 \rangle$

3. Solve the differential equation $2y' = y^{-1}e^{-t}$.
    A. $2y^2 = ye^t$
    B. $y^2 - ye^{-t} = 0$
    C. $2y - e^{2t} = 1$
    D. $y^2 + e^{-t} = 0$

4. Which of the following statements is/are true about vectors A, B and C?

   I. $(A + B) \times C = (A \times C) + (B \times C)$
   II. $A \times B = B \times A$
   III. $A \cdot B = B \cdot A$

   A. I only
   B. I and III only
   C. II and III only
   D. All the above.

5. Determine the degree of homogeneity of the function $f(x,y) = \dfrac{2x+y}{x^3 y^3}$.

   A. 4
   B. -2
   C. 6
   D. -5

6. Find the value of λ for which vector $U = 2i + \lambda j + 6k$ and vector $V = i + 2j + 3k$ are parallel.
   A. $\lambda = 4$
   B. $\lambda = -2$
   C. $\lambda = 5$
   D. $\lambda = 1$

7. Which of the following differential equations satisfies $y^2 - x^2 = 10 + x$?
   A. $y' = 1 + 2x$
   B. $2yy' = 1 + 2x$
   C. $y' = 10 + 2x + y$
   D. $y' = 1 + 2x^2$

8. Which of the following statements best describes coplanar vectors? They ....
   A. lie on opposite planes.
   B. lie on different planes.
   C. lie on the same plane.
   D. are equal.

9. Given two vectors a and b, if the angle between a and b is zero, then ....
   A. $a \cdot b = |a||b|$
   B. $a \cdot b = 0$
   C. $a = b$
   D. None of the above.

10. Find the integrating factor of the equation $\dfrac{dy}{dx} = 5x - \dfrac{3y}{x}$

   A. x
   B. x^3
   C. y
   D. y^3

11. Solve the differential equation $\dfrac{dy}{dx} = \dfrac{6x^2}{2y + \cos y}$

    A. $y^2 + \sin y - 2x^2 - c = 0$

    B. $y^2 - \cos y + 3x^2 + c = 0$

    C. $y^2 - \tan y - x^2 + c = 0$

    D. $y^2 - 2x^2 + c = 0$

12. Which of the following is a vector quantity?

    A. Moment.

    B. Density.

    C. Magnification.

    D. Energy.

13. Given two vectors $A = 2i + 2j - k$ and $B = 6i - 3j + 2k$. Find $\cos\theta$; where θ is the angle between vector A and B.

    A. $\dfrac{4}{21}$

    B. 4

    C. 0.5

    D. 1

14. Find the unit vector in the direction of the sum of the vectors $P = 2i - j + 2k$ and $Q = -i + j + 3k$

    A. $\dfrac{1}{\sqrt{26}}i + 5k$

    B. $5i - 5k$

    C. $\dfrac{1}{\sqrt{26}}i + \dfrac{5}{\sqrt{26}}k$

    D. $-5i + 5k$

15. What is the degree of the differential equation $\left(\dfrac{d^3y}{dx^3}\right)^2 + \left(\dfrac{d^2y}{dx^2}\right)^5 + \dfrac{y}{x^2 + 1} = e^x$

    A. 2

    B. 3

    C. 4

    D. 5

16. If vector $A = 3i - j - 4k$, $B = -2i + 4j - 3k$ and $C = i + 2j - k$, find $|A + B + C|$.

    A. $\sqrt{93}$

    B. $9\sqrt{2}$

    C. $-5\sqrt{3}$

    D. 10

17. If $a = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ find the magnitude of the unit vector $\hat{a}$.

    A. $\sqrt{14}$

    B. $\dfrac{2}{5}$

    C. 0

    D. 1

18. The following are examples of a linear differential equation **except** ....

    A. $\dfrac{dy}{dx} + y = x^2$

    B. $3x^2 y'' + (2\ln x)y' + e^x y = 3x\cos x$

    C. $4yy'' - x^3 y' + \cos y = e^{2x}$

    D. $y' + 3y = 0$

19. Which of these is a solution to the differential equation $y'' + y = 0$

    A. $y = e^{3x}$

    B. $y = 2x$

    C. $y = \sin x$

    D. $\ln y = A\cos x + B\sin x$

20. For exact differential equation of the form $M(x,y)dx + N(x,y)dy = 0$

    A. $\dfrac{\partial M}{\partial x} = \dfrac{\partial N}{\partial y}$

    B. $\dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} \neq 0$

    C. $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$

    D. $\dfrac{\partial M}{\partial y} + \dfrac{\partial N}{\partial x} = 0$

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SECTION B

Answer any TWO questions from this Section.

1.

   a. Given the initial value problem $2xyy' = y^2 - x^2$; $y(1) = 3$. Show that $y^2 + x^2 = 10x$.
      Hint: (set $u = \frac{y}{x}$).

   b. The coordinates of three points in a three-dimensional vector space are $P(-2,5,7)$, $Q(7,3,-3)$
      and $R(2,-1,6)$.
      i. Calculate the area of the triangle obtained by connecting the three points P, Q and R by
         straight lines.
      ii. Find the unit vector in the direction of vector $\overrightarrow{PQ} + \overrightarrow{PR}$

2.

   a. Solve the homogenous differential equation $\dfrac{dy}{dx} = \dfrac{x^2 + y^2}{xy}$.

   b. i. Find the particular solution of the IVP $xy' = -y$, for $x > 0$ such that $y(4) = 2$.

      ii. Determine whether or not the two vectors $A = 2i + j + 3k$ and $B = 6i + 3j + 9k$
          are parallel.

3.

a. Reduce the differential equation $y' + 2y = y^2$ to linear form and solve its particular solution given that; $y(0) = \dfrac{2}{5}$.

b. Given two vectors $U = 4i + 3j + 2k$ and $V = 8i - 6j$. Find

  i. $|U|^2$ and the angle between vector U and vector $\frac{1}{2}V$.

  ii. all the direction angles of vector V.

4.

a. Solve the initial value problem $y' + 5y = -10xy^2$; $y(0) = 1$.

b. i. If $A = 2i + j - 3k$ and $B = i - 2j + k$, Find a vector of magnitude 10 and perpendicular to both A and B.

  ii. Define scalar multiplication of a vector.