JANUARY 2023
MAT 405SW
ORDINARY DIFFERENTIAL EQUATIONS
2 HOURS

Candidate's Index Number
Signature:

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
SCHOOL OF EDUCATIONAL DEVELOPMENT AND OUTREACH
INSTITUTE OF EDUCATION

FIVE-SEMESTER BACHELOR OF EDUCATION (SANDWICH) PROGRAMME
LEVEL 400, END-OF-FIRST SEMESTER EXAMINATIONS, JANUARY 2023

5TH JANUARY 2023 ORDINARY DIFFERENTIAL EQUATIONS 2:00 PM - 2:40 AM

This paper consists of two sections, A and B. Answer ALL the questions in Section A and TWO questions from Section B. Section A will be collected after the first 40 minutes.

SECTION A
(20 Marks)

Answer ALL the questions in this Section.

Items 1 to 20 are stems followed by four options lettered A to D. Read each item carefully and circle the letter of the correct or best option.

1. Given $y_1 = e^{2t}$ and $y_2 = te^{2t}$, which of the following statements is true about the Wronskian $W(y_1, y_2)$?
   A. e^{4t}, therefore y_1 and y_2 are linearly independent.
   B. e^{4t}, therefore y_1 and y_2 are linearly dependent.
   C. 0, therefore y_1 and y_2 are linearly independent.
   D. e^{-4t}, therefore y_1 and y_2 are linearly dependent.

2. For which values of m is the function $y = x^m$ a solution of the differential equation $x^2y'' - 5xy' + 8y = 0$?
   A. $m = 2, 3$
   B. $m = 2, 4$
   C. $m = 3, 4$
   D. $m = -2, -4$

3. Given $y'' - 4y' + 3y = e^t + t^2$, find the particular solution.
   A. $y_p = Ae^t + Bt^2$
   B. $y_p = Ae^t + Bt^2 + Ct + D$
   C. $y_p = Ate^t + Bt^2$
   D. $y_p = Ate^t + Bt^2 + Ct + D$

A. $e^y = C(e^x + 1)(y + 1)$

B. $e^y = C(e^x + y + 1)$

C. $y = C(e^x + 1)(y + 1)$

D. $e^x = C(e^x + 1)(y + 1)$

5. What is the solution to initial value problem $y' = y(2x + 1), y(0) = 2$?

A. $y = 2e^{x(x+1)}$

B. $y = \frac{2\ln(y)}{x^2 + \ln 2}$

C. $y = 2e^{x(2x+1)}$

D. $y = e^{x^2} + e^x$

6. The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ is,

A. 2

B. 3

C. 1

D. Undefined

7. Solve for the integrating factor of $y' = ay + b$.
   A. $\mu(t) = e^{-at}$
   B. $\mu(t) = e^{at}$
   C. $\mu(t) = e^{t}$
   D. $\mu(t) = e^{-t}$.

8. Find a second solution y_2 linearly independent to the solution $y_1(t) = t$ of the differential equation $t^2y'' + 2ty' = 0$.
   A. $y_2 = t^{-3}$
   B. $y_2 = t^{-2}$
   C. $y_2 = t^2$
   D. $y_2 = t^{-1}$

9. Which of the following is not true about Bernoulli equation $y' = p(t)y + q(t)ty^n$?
   A. p, q are given functions.
   B. For $n \neq 0, 1$ the equation is nonlinear.
   C. For $n \neq 0, 1$ the equation is linear.
   D. It is a nonlinear equation that can be transform into linear equation.

10. Given $y_1(t) = e^t$ and $y_2(t) = te^t$ as a fundamental set of solutions to a homogeneous second order linear differential equation, find the other pair which does not constitute a fundamental set of solutions to the same differential equation.

A. $y_3(t) = e^{t+3}$ and $y_4(t) = e^{t-2}$
B. $y_3(t) = e^t$ and $y_4(t) = (t+1)e^t$
C. $y_3(t) = e^{t+1}$ and $y_4(t) = (t+1)e^{t+1}$
D. $y_1(t) = 2e^t$ and $y_2(t) = -te^t$

11. What is the order of the differential equation $\frac{d^2y}{dx^2} - 2\left(\frac{dy}{dx}\right)^5 - 8y = x^4$?

A. 5
B. 4
C. 3
D. 2

12. The general solution to $y' = xy^2$ has the form

A. $y = \pm\sqrt{e^{\frac{x^2}{2}+c}}$.

B. $y = \frac{x^2y^2}{2} + c$

C. $y = \frac{-2}{x^2+c}$

D. $y = \frac{x^2y^3}{6} + c$

13. Find an integrating factor for $y' + \left(\frac{4}{t}\right)y = t^4$.

A. $\ln t^4$
B. t^4
C. t^{-4}
D. t^3

14. To solve the equation $3\frac{dy}{dx} + 5y^2 = \sin x, y(0) = 5$ by Euler's method, you need to rewrite the equation as

A. $\frac{dy}{dx} = \sin x - 5y^2, y(0) = 5.$

B. $\frac{dy}{dx} = \frac{1}{3}(\sin x - 5y^2), y(0) = 5.$

C. $\frac{dy}{dx} = \frac{1}{3}\left(-\cos x - \frac{5y^2}{3}\right), y(0) = 5.$

D. $\frac{dy}{dx} = \frac{1}{3}\sin x, y(0) = 5.$

15. Solve $y' - 5y = 0$.

A. e^{5t}

B. e^{-5t}

C. Ce^{5t}

D. Ce^{-5t}

A. $C_2t^4 + C_1t^3 + C_0t$

B. $C_2t^2 + C_1t + C_0$

C. $C_2t^3 + C_1t^2 + C_0$

D. $C_2t^3 + C_1t^2 + C_0t$

17. The roots of the auxiliary equation of $y'' + y' - 2y = 0$ are
    A. $-1, 2$
    B. $1, -2$
    C. $-2, 2$
    D. $-1, 1$

18. Find general solution of the equation $y'' + 6y' + 9y = 0$.
    A. $y(t) = c_1e^{-3t} + c_2te^{-3t}$
    B. $y(t) = c_1e^{-3t} + c_2e^{-3t}$
    C. $y(t) = c_1e^{-3t} + c_2te^{3t}$
    D. $y(t) = c_1e^{3t} + c_2e^{-3t}$

19. Given $y_1 = x$ and $y_2 = \frac{1}{x}$, find the Wronskian determinant.

    A. $-\frac{1}{x}$
    B. $\frac{1}{x}$
    C. $-\frac{2}{x}$
    D. $\frac{2}{x}$

20. What is the choice of the particular solution for the nonhomogeneous equation $y'' + ay' + by = Ke^{\alpha x}$?
    A. $y_p = Ce^{\alpha x}$
    B. $y_p = e^{\alpha x}$
    C. $y_p = Ce^{-\alpha x}$
    D. $y_p = e^{-\alpha x}$

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TH JANUARY 2023 **ORDINARY DIFFERENTIAL EQUATIONS** **2:40 PM - 4:00 AM**

SECTION B

Answer any TWO questions from this section. [Each questions carries 15 marks]

1.

a. By the reduction of order method, solve $y'' - 5y = 0$.

b. Find the differential equation that is satisfied by the family of functions $y(x) = c_1 x + c_2 x^2$, where c_1, c_2 are arbitrary constants.

2.

a. Find the general solution of $y'' + 2y' + y = 2e^{-t}$.

b. Solve differential equation $y' + \frac{y}{x} = \cos(x)$

3.

a. Solve $\frac{dy}{dx} + \frac{1}{3}y = e^x y^4$

b. If $y_1 = x^2$ is a solution to $x^2 y'' - 3xy' + 4y = 0$, find the general solution on the interval $(0, \infty)$.

4.

a. Prove that the Euler homogeneous equation $y' = F\left(\frac{y}{t}\right)$ for the function y determines a separable equation for $v = \frac{y}{t}$, given by

$$\frac{v'}{(F(v) - v)} = \frac{1}{t}$$

b. Find the general solution of the equation $ty' = 3y + t^5 y^{\frac{1}{3}}$.