Atanga. Rita A

IE/MAT/SIJ/22/0014

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INSTITUTE OF EDUCATION

MAT 201SW: INTRODUCTION TO ABSTRACT ALGEBRA

MARKING SCHEME (2020)

SECTION A

1. D
2. A
3. C
4. D
5. C
6. B
7. C
8. A
9. B
10. A
11. B
12. A
13. D
14. D
15. C
16. B
17. D
18. B
19. C
20. A

SECTION B

Q1. (a) Let $P_n: 1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{1}{4}n^2(n+1)^2$ for all $n \in \mathbb{N}$.

[B1]

For $n = 1$,

LHS of $P_1: 1^3 = 1$ [M1]

RHS of $P_1: \frac{1}{4}(1^2)(1+1)^2 = \frac{4}{4} = 1$ [M1]

Thus, P_1 is true. [A1/2]

Assume that

4. (a) Determine whether $\propto : \mathbb{R} \longrightarrow \mathbb{R}$ defined by

$$\propto (x) = \sqrt{x} \quad \text{for all } x \in \mathbb{R}$$

is a mapping. Justify your answer.

(b) Let $\mathbb{Z}$ be the set of all integers and $f : \mathbb{Z} \dashrightarrow \mathbb{Z}$ be defined by

$$f(x) = 2x + 1, x \in \mathbb{Z}.$$

Determine whether f is

(i) a one-to-one mapping.

(ii) an onto mapping.

AUGUST 2020
MAT 201SW
INTRODUCTION TO ABSTRACT ALGEBRA
1 HOUR 30 MINUTES

Candidate's Index Number
Signature:

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
SCHOOL OF EDUCATIONAL DEVELOPMENT AND OUTREACH
INSTITUTE OF EDUCATION

FIVE-SEMESTER BACHELOR OF EDUCATION (SANDWICH) PROGRAMME
FIRST YEAR, END-OF-SECOND SEMESTER EXAMINATION, AUGUST 2020

AUGUST 29, 2020 INTRODUCTION TO ABSTRACT ALGEBRA 9:30 AM - 11:00 AM

SECTION B

Answer any THREE questions from this section.

1. (a) Prove by mathematical induction that
$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{1}{4}n^2(n+1)^2$$
for all $n \in \mathbb{N}$.

   (b) If $a, b \in \mathbb{Z}$, prove that $(-a) + (-b) = -(a+b)$.

2. (a) Show that the operation $a * b = 1 + ab$ defined on the set of integers $\mathbb{Z}$ has no identity element.

   (b) Let $B = \mathbb{R} - \{0\}$. Define the relation $\mathcal{R}$ on B by $a\mathcal{R}b$ if and only if $ab > 0$. Show that $\mathcal{R}$ is an equivalence relation on B.

3. (a) Let A, B and C be sets. Prove that
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

   (b) Show that if $x = y + z$ and d is a divisor of any two of the integers x, y and z, it is also a divisor of the third.

16. Express gcd(72, 26) in the form $72m + 26n$.

A. $72(4) + 26(-11)$

B. $72(4) + 26(6)$

C. $72(8) + 26(11)$

D. $72(-2) + 26(11)$

Items 17 to 20 are statements followed by True and False option. Read each state carefully and indicate whether the statement is True or False, by circling the letter o correct option.

17. The mapping $f: \mathbb{N} \to \mathbb{N}$ defined by $f(x) = x + 3$ is one-to-one.

A. True

B. False

18. The mapping $g: \mathbb{N} \to \mathbb{Z}$ defined by $g(x) = x^2 - x$ for all $x \in \mathbb{N}$ is onto.

A. True

B. False

19. The mapping $h: [3, \infty) \to [5, \infty)$ defined by $h(x) = (x - 3)^2 + 5$ is one-to-on and onto.

A. True

B. False

20. The relation

$$R = \{(1,1), (1,2), (2,2), (3,1), (3,3), (2,1), (2,3), (1,3), (3,2)\}$$

on the set $A = \{1,2,3\}$ is an equivalence relation.

A. True

B. False

9. Which of the following relations is not reflexive?
   A. aRb if and only if $a \leq b, a, b \in \mathbb{Z}$
   B. ARB if and only if $A \subseteq B, A, B \in P(S)$
   C. aRb if and only if $a \equiv b \ (mod\ n),\ \ a, b \in \mathbb{Z}$
   D. aRb if and only if $a + b \equiv 0 \ (mod\ n)\ ,\ \ a, b \in \mathbb{Z}$

10. Which of the following is a mapping?
   A. $\propto: \mathbb{N} \longrightarrow \mathbb{N}$ defined by $\propto(n) = (-1)^n n^2$ for all $n \in \mathbb{N}$.
   B. $\propto: \mathbb{R} \longrightarrow \mathbb{R}$ defined by $\propto(x) = 2 + \sqrt{x}$ for all $x \in \mathbb{R}$.
   C. $\alpha: \{e, f\} \rightarrow \{2, 7, 12\}$ defined by $\{(e, 7), (f, 2), (f, 12)\}$.
   D. $\alpha: \mathbb{Z} \rightarrow \mathbb{Z}$ defined by $\alpha(k) = k^2 + k$ for all $k \in \mathbb{Z}$.

11. Let $a, b \in \mathbb{Z}^+$ such that $b \nmid a$ and b is a prime. Find $\gcd(a, b)$.
   A. a
   B. b
   C. 1
   D. 2

12. If $m = kn$ for $m, n, k \in \mathbb{Z}^+$, what is $\gcd(m, n)$?
   A. m
   B. n
   C. k
   D. 1

13. If there exist integers m and n such that $ma + nb - 7 = 0$, find $\gcd(a, b)$.
   A. 1
   B. m
   C. -7
   D. 7

14. Find $[s, m].[t, n]$ if $[s, m] = [2, 3]$ and $[t, n] = [1,4]$.
   A. $[3, 7]$
   B. $[14, 11]$
   C. $[6, 4]$
   D. $[2, 12]$

15. Given that $[3] = \{3, 6, 9, 12, 15, 18, 21\}$. Find $[18]$.
   A. $\{18, 21\}$
   B. $\{12, 15, 18, 21\}$
   C. $\{3, 6, 9, 12, 15, 18, 21\}$
   D. $\{3, 12, 18\}$

4. A binary operation Δ on the set A of positive real numbers is given by $a\Delta b =$ $\sqrt{ab}$. Find the identity element for Δ if it exists.

A. $1/a$

B. 1

C. 4

D. Does not exist

Use the following information to answer questions 5 to 7.

Suppose that the set A has as exactly two elements and B has exactly three elem...

5. How many mappings are there from B to A?

A. 4

B. 5

C. 8

D. 9

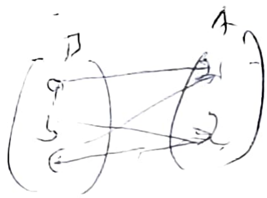
6. How many one-to-one mappings are there from B to A?

A. 0

B. 2

C. 6

D. 9



7. How many onto mappings are there from B to A?

A. 0

B. 2

C. 6

D. 9

8. An equivalence relation R on $\mathbb{N}$ is defined by aRb if $a^2 + b^2$ is even. Find $[1]$

A. $\{n \in \mathbb{N} : n \text{ is even}\}$

B. $\{n \in \mathbb{N} : n \text{ is odd}\}$

C. $\{1, 2, 3, 4\}$

D. $\{1,3,5,7\}$

$$P_k: 1^3 + 2^3 + 3^3 + \cdots + k^3 = \frac{1}{4}k^2(k+1)^2$$

is true. [M1]

Then,

$$1^3 + 2^3 + 3^3 + \cdots + k^3 + (k+1)^3 = \frac{1}{4}k^2(k+1)^2 + (k+1)^3 \quad \text{[M1 M1]}$$

$$= \frac{1}{4}(k+1)^2[k^2 + 4(k+1)] \quad \text{[M1]}$$

$$= \frac{1}{4}(k+1)^2[k^2 + 4k + 4] \quad \text{[M1]}$$

$$= \frac{1}{4}(k+1)^2(k+2)^2 \quad \text{[M1]}$$

$$= P_{k+1} \quad \text{[M1]}$$

Thus, P_{k+1} is true. [M1/2]

Therefore, the statement P_n is true for all $n \in \mathbb{N}$. [A1]

(b) Let $a \leftrightarrow [s, m]$ and $b \leftrightarrow [t, n]$. [B1/2 B1/2]

Then

$$-a \leftrightarrow [m, s] \quad \text{[M1]}$$

and

$$-b \leftrightarrow [n, t] \quad \text{[M1]}$$

Thus,

$$(-a) + (-b) \leftrightarrow [m, s] + [n, t] = [(m+n), (s+t)] \quad \cdots \ (1) \quad \text{[M1]}$$

Also,

$$a + b \leftrightarrow [s, m] + [t, n] = [(s+t), (m+n)] \quad \text{[M1 M1]}$$

Then

$$-(a+b) \leftrightarrow [(m+n), (s+t)] \quad \cdots \ (2) \quad \text{[M1]}$$

From (1) and (2) we have that

$$(-a) + (-b) = -(a+b) \quad \text{[A1]}$$

Q2. (a) Suppose e is the identity element for $*$. [M1]

Then

$$a * e = a \quad [M1]$$

$$\implies 1 + ae = a \quad [M1]$$

$$\implies ae = a - 1 \quad [M1]$$

$$\implies e = 1 - \frac{1}{a} \quad [M1]$$

Since e depends on a, it means that e is not unique. [M1 A1]

Hence, $*$ has no identity element. [A1]

(b) Let $x \in B$. [M1/2]

Then $xx = x^2 > 0$, that is, [M1/2]

$$x\mathcal{R}x \text{ for all } x \in B. \quad [M1/2]$$

Thus, $\mathcal{R}$ is reflexive. [M1/2]

Suppose $x, y \in B$ and $x\mathcal{R}y$, that is $xy > 0$. [M1/2]

Since $xy = yx$ we have that $yx > 0$ and so $y\mathcal{R}x$. [M1/2]

Thus $\mathcal{R}$ is symmetric. [M1/2]

Suppose $x, y, z \in B$ such that $x\mathcal{R}y$ and $y\mathcal{R}z$, that is, [M1/2]

$$xy > 0 \text{ and } yz > 0 \quad [M1/2\ M1/2]$$

If $x > 0$, then since $xy > 0$ we have $y > 0$. [M1/2 M1/2]

Since $y > 0$ and $yz > 0$ then $z > 0$. [M1/2 M1/2]

Since $x > 0$ and $z > 0$ it follows that $xz > 0$, that is, $x\mathcal{R}z$. [M1/2 M1/2]

Also if $x < 0$, then since $xy > 0$, then $y < 0$. [M1/2 M1/2]

Since $y < 0$ and $yz > 0$ then $z < 0$. [M1/2 M1/2]

AUGUST 2020
MAT 201SW
INTRODUCTION TO ABSTRACT ALGEBRA
2 HOURS

Candidate's Index Number
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UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
SCHOOL OF EDUCATIONAL DEVELOPMENT AND OUTREACH
INSTITUTE OF EDUCATION

FIVE-SEMESTER BACHELOR OF EDUCATION (SANDWICH) PROGRAMME
FIRST YEAR, END-OF-SECOND SEMESTER EXAMINATION, AUGUST 2020

AUGUST 29, 2020 INTRODUCTION TO ABSTRACT ALGEBRA 9:00 AM - 9:30 AM

This paper consists of two sections, A and B. Answer all the questions in Section A and THREE questions in Section B. Section A will be collected after the first 30 minutes.

SECTION A

Answer all the questions in this section

For items 1 to 16, each stem is followed by four options lettered A to D. Read each item carefully and circle the letter that corresponds to the correct or the best option

1. Let P and Q be sets. Which of the following is true if $P - Q = Q - P$?
   A. $P \cap Q = \phi$
   B. $P \subseteq Q$
   C. $Q \subseteq P$
   D. $P = Q$

2. Which of the following is not true about a set Q and its universal set U?
   A. $Q \cap \phi = Q$
   B. $Q \cap Q' = \phi$
   C. $Q \cap U = Q$
   D. $Q \cup Q' = U$

3. Which of the following is not a binary operation on the given set. (where the set notation maintain their normal meaning)
   A. $*$ defined on $\mathbb{Z}$ by $a * b = b + a$
   B. $*$ defined on $\mathbb{Z}_n$ by $[a] * [b] = [a] \odot [b]$
   C. $*$ defined on $S = \{1, -2, 3, 2, -4\}$ by $a * b = |b|$
   D. $*$ defined on $\mathbb{Z}$ by $a * b = a^b$

$\Rightarrow a + e - 2(ae)^2 = a$[M1/2]

$\Rightarrow e - 2(ae)^2 = 0$

$\Rightarrow e(1 - 2a^2 e) = 0$[M1/2]

$\Rightarrow e = 0$ [M1/2]

or

$1 - 2a^2 e = 0 \implies e = \frac{1}{2a^2}$[M1/2]

Therefore, $*$ has no identity element sincee is not unique.[A1/2]

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Q3.Suppose that $p|q$and $p|r$. Then there exist integers m and n such that

$q = np$ ... (1) [M1/2]

and

$r = mp$... (2) [M1/2]

Multiplying equations (1) and (2) by x and y respectively gives

$qx = npx$and$ry = mpy$[M1/2 M1/2]

Thus,

$qx + ry = npx + mpy$[M1]

$\implies qx + ry = p(nx + my)$[M1/2]

5

$\Rightarrow p|(qx + ry)$since$nx + my \in \mathbb{Z}$[M1 A1/2]

Q4.By the Euclidean Algorithm we have that

$726 = 275(2) + 176$[M1]

$275 = 176(1) + 99$[M1]

$176 = 99(1) + 77$[M1/2]

$99 = 77(1) + 22$[M1/2]

$77 = 22(3) + 11$[M1/2]

$22 = 11(2)$[M1/2]

Thus, $\gcd(726, 275) = 11$. [A1]

5

INSTITUTE OF EDUCATION

MAT 201: Introduction to Abstract Algebra

QUIZ 2 Marking Scheme (2019)

Q1. Let $P_n: \sum_{k=1}^{n}(2k-1) = n^2$ for all $n \geq 1$. [B1/2]

For $n = 1$,

LHS of $P_1: 2(1) - 1 = 1$

RHS of $P_1: 1^2 = 1$

Thus, P_1 is true.[M1/2 A1/2]

Assume that

$$P_n: \sum_{k=1}^{n}(2k-1) = n^2$$

is true.[M1/2]

Then,

$\sum_{k=1}^{n+1}(2k-1) = \sum_{k=1}^{n}(2k-1) + 2(n+1) - 1$[M1/2]

$= n^2 + 2(n+1) - 1$[M1/2]

$= n^2 + 2n + 2 - 1$

$= n^2 + 2n + 1$

$= (n+1)^2$**[M1/2]**

$= P_{k+1}$[M1/2]

Thus, P_{k+1} is true. [M1/2]

Therefore, the statement P_n is true for all $n \geq 1$.[A1/2]

Q2. (i) By definition

$a * b = a + b - 2(ab)^2$ ... (1)[B1/2]

But addition of real numbers is commutative so $a + b = b + a$.

Thus equation (1) is the same as,

$b * a = b + a - 2(ab)^2$... (2)[M1/2]

Also, multiplication of real numbers is commutative so $ab = ba$.

Thus equation (2) becomes

$a * b = b + a - 2(ba)^2 = b * a$[M1/2]

Therefore, $*$ is commutative since $a * b = b * a$.[A1/2]

(ii) Let e denote the identity element of $*$. Thus,

$a * e = a$[M1/2]

Candidate's Index No.
IE/MMT/ARR/1...
Signature: K...

SEPTEMBER 2019
MAT 201SW
INTRODUCTION TO
ABSTRACT ALGEBRA
1 HOUR

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
SCHOOL OF EDUCATIONAL DEVELOPMENT AND OUTREACH
INSTITUTE OF EDUCATION

FIVE-SEMESTER BACHELOR OF EDUCATION (SANDWICH) PROGRAMME
FIRST YEAR, FIRST SEMESTER QUIZ II, SEPTEMBER 2019

SEPTEMBER 1, 2019 INTRODUCTION TO ABSTRACT ALGEBRA 1:30 PM

Answer All the questions

1. Prove by mathematical induction that

$$\sum_{k=1}^{n}(2k-1)=n^2$$

for $n \geq 1$.

2. Let $*$ be a binary operation defined on the set of real numbers $\mathbb{R}$ by
$a * b = a + b - 2(ab)^2$, where $a, b \in \mathbb{R}$.

 (i) Prove that $*$ is commutative.

 (ii) Find the identity element for $*$.

3. If $p|q$ and $p|r$, prove that $p|(qx + ry)$ where x and y are integers.

4. Find $\gcd(726, 275)$ using the Euclidean Algorithm.

Thus,

$$x \in A \cap (B \cup C) \quad [M1]$$

$$\Rightarrow (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C) \quad [A1]$$

Therefore,

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \quad [A1]$$

(b) Suppose that

$$d|y \text{ and } d|z \quad [B1\ B1]$$

Then there are integers m and n such that

$$y = md \quad [M1]$$

and

$$z = nd \quad [M1]$$

But

$$x = y + z \quad [M1]$$

$$\Rightarrow x = md + nd \quad [M1]$$

$$\Rightarrow x = (m + n)d \quad [M1]$$

$$\Rightarrow d|x \text{ since } m + n \in \mathbb{Z}. \quad [A1]$$

Q4. (a) $\alpha(x)$ is not a mapping since [M2]

$$\alpha(-1) = \sqrt{-1} \notin \mathbb{R} \quad [A2]$$

(b) (i) Suppose that $x_1, x_2 \in \mathbb{Z}$ such that $f(x_1) = f(x_2)$ [B2]

Then

$$f(x_1) = f(x_2) \quad [M1]$$

Since $x < 0$ and $z < 0$ we have $xz > 0$ that is, $x\mathcal{R}z$. [M1/2 M1/2]

Therefore, $\mathcal{R}$ is transitive. [M1/2]

Hence, $\mathcal{R}$ is an equivalence relation. [A1/2]

Q3. (a) Let $x \in A \cap (B \cup C)$. [M1/2]

Then

$x \in A$ and $x \in B \cup C$ [M1]

$\Rightarrow x \in A$ and $x \in B$ or $x \in C$ [M1/2]

$\Rightarrow x \in A$ and $x \in B$ or $x \in A$ and $x \in C$ [M1]

When

$x \in A$ and $x \in B$, then $x \in A \cap B$ so $x \in (A \cap B) \cup (A \cap C)$

[M1/2]

Also, when

$x \in A$ and $x \in C$ then $x \in A \cap C$ so $x \in (A \cap B) \cup (A \cap C)$

[M1/2]

Thus,

$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ [A1]

Now, let $x \in (A \cap B) \cup (A \cap C)$. [M1/2]

Then

$x \in A \cap B$ or $x \in A \cap C$ [M1]

When $x \in A \cap B$ then $x \in A$ and $x \in B$ [M1/2]

$\Rightarrow x \in A$ and $x \in B \cup C$ [M1/2]

Also, when $x \in A \cap C$ then $x \in A$ and $x \in C$ [M1/2]

$\Rightarrow x \in A$ and $x \in B \cup C$ [M1]

SEPTEMBER 2018
MAT 201S
INTRODUCTION TO ABSTRACT ALGEBRA
2 HOURS

Candidate's Index Number
Signature:

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
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INSTITUTE OF EDUCATION

FIVE-SEMESTER BACHELOR OF EDUCATION (SANDWICH) PROGRAMME
FIRST YEAR, END-OF-FIRST SEMESTER EXAMINATION, SEPTEMBER 2018

SEPTEMBER 7, 2018 INTRODUCTION TO ABSTRACT ALGEBRA 9:00 AM–9:40 AM

This paper consists of two sections, A and B. Answer all the questions in Section A and Three questions in Section B. Section A will be collected after the first 40 minutes.

SECTION A
Answer ALL the questions in this section.

For items 1 to 20, each stem is followed by four options lettered A to D. Read each statement carefully and circle the letter that corresponds to the correct or best option.

1. Which of the following is not true about the sets P, Q and R?
   A. $P - Q = P \cap Q'$
   B. $(P \cup Q)' = P' \cap Q'$
   C. $(P \cap Q)' = P' \cup Q'$
   D. $P - (Q \cap R) = (P - Q) \cap C$

2. Which of the following is a mapping?
   A. $\alpha: \{1, 2, 3\} \to \{a, b, c\}$ defined by $\{(1, a), (3, b)\}$.
   B. $\alpha: \mathbb{R} \to \mathbb{R}$ defined by $\alpha(x) = \sqrt{x}$ for all $x \in \mathbb{R}$.
   C. $\alpha: \mathbb{N} \to \mathbb{N}$ defined by $\alpha(n) = -n$ for all $n \in \mathbb{N}$.
   D. $\alpha: \{1, 2, 3\} \to \{0, 3, 8\}$ defined by $\{(1, 0), (2, 3), (3, 8)\}$.

3. The mapping $\sigma: \mathbb{R} \to \mathbb{R}$ defined by $\sigma(x) = x^3 - 3x^2 - x$ is one-to-one.
   A. True
   B. False

$$\Rightarrow 2x_1 + 1 = 2x_2 + 1 \quad [M2]$$

$$\Rightarrow 2x_1 = 2x_2 \quad [M1]$$

$$\Rightarrow x_1 = x_2 \quad [M1]$$

Thus, $f(x)$ is one-to-one. [A2]

(ii) Let $y_1 \in \mathbb{Z}\ (co - domain)$ and $x_1 \in \mathbb{Z}\ (domain)$ [B1 B1]

Then $2x_1 + 1 = y_1$ is odd for every integer x_1 [M2]

Since there is no such integer x_1 if y_1 is even $f(x)$ is not onto. [M2 A1]

4. The mapping $\gamma: \mathbb{N} \to \mathbb{N}$ defined by $\gamma(x) = x + 2$ for all $x \in \mathbb{N}$ is onto.

A. True

B. False

5. Let $\alpha: S \to T$ and $\beta: T \to U$ be two mappings. If α and β are invertible with inverses ρ and δ respectively, find $(\beta\alpha)^{-1}$.

A. ρ

B. $\delta\rho$

C. $\rho\delta$

D. δ

6. Let $E = \{1, 2, 3\}$ and $F = \{x, y\}$. Find the number of mappings from F into E.

A. 9

B. 8

C. 3

D. 1

7. Which of the following is a binary operation on the given set.

A. $*$ defined on Z^+ by $a * b = a - b$

B. $*$ defined on Z by $a * b = a/b$

C. $*$ defined on Z by $a * b = a + b$

D. $*$ defined on Z by $a * b = \frac{1}{3}(a + b)$

Use the following information to answer questions 8 and 9.

A binary operation Δ on the set U of ordered pairs (p, q) of real numbers is given by $(a, b)\Delta(c, d) = (ac - bd, ad + bc)$.

8. Find the identity element in U under Δ.

A. 0

B. (0, 0)

C. (1, 0)

D. (0, 1)

9. Find the inverse of (a, b) under Δ.

A. $\left(\frac{-a}{a^2+b^2}, \frac{b}{a^2+b^2}\right)$

B. $\left(\frac{a}{a^2+b^2}, \frac{b}{a^2+b^2}\right)$

C. $\left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$

D. $\left(\frac{-b}{a^2+b^2}, \frac{a}{a^2+b^2}\right)$

Use the following information to answer questions 10 and 11.

Define the relation R on $V = \{-2, -1, 0, 1, 2\}$ by

$R = \{(a, b): a^3 - a = b^3 - b\}$.

10. Which of the following elements does not belong to R?

A. $(2, 2)$
B. $(-2, 1)$
C. $(-2, -2)$
D. $(1, -1)$

11. Find the equivalence class of $[1]$.

A. $\{-1, 1, 2\}$
B. $\{-1, 0, 1, 2\}$
C. $\{-2, -1, 0, 1, 2\}$
D. $\{-1, 0, 1\}$

12. Which of the following relations is not reflexive?

A. aRb if and only if $a \leq b, a, b \in \mathbb{Z}$
B. ARB if and only if $A \subseteq B, A, B \in P(S)$
C. aRb if and only if $a - b$ is divisible by 4, $a, b \in \mathbb{Z}$
D. aRb if and only if $ab = 1$, $a, b \in \{x \in \mathbb{R} \mid x > 0\}$

13. Define the relation R on the set of real numbers by aRb if and only if $ab \geq 0$. Determine all the properties of a partial ordering which R does not have.

A. transitive
B. anti-symmetric, transitive
C. reflexive, anti-symmetric, and transitive
D. anti-symmetric, and reflexive

14. Define on the set $\mathbb{N}$ the relation "aRb if and only if $b = ka$" for some integer k. Find the properties of an equivalence relation which R does not have.

A. symmetric
B. reflexive and transitive
C. symmetric and transitive
D. reflexive and symmetric

Use the following information to answer questions 15 and 16.

Define the relation R on $A = \mathbb{N} \times \mathbb{N}$ by $(a, b)R(c, d)$ if and only if $ab = cd$.

15. Find an equivalence class with exactly two elements.

A. $[(4, 5)]$
B. $[(6,3)]$
C. $[(9, 2)]$
D. $[(1, 2)]$

16. Find an equivalence class with exactly three elements.

A. $[(1,4)]$
B. $[(6, 3)]$
C. $[(1, 18)]$
D. $[(9, 2)]$

17. If $[s, m], [t, n] \in J$, find $[s, m] + [t, n]$.

A. $[(s + m), (t + n)]$
B. $[(s + t), (m + n)]$
C. $[(s + n), (m + t)]$
D. $[(sm), (tn)]$

18. Which of the following is true about (c, d) and (s, m) if $(c, d) \in [s, m]$?

A. $[c, d] = [s, m]$
B. $(c, d) = (s, m)$
C. $cd = sm$
D. $[c + s, d + m]$

19. Find $(1776, 1492)$, that is, the greatest common divisor of 1776 and 1492.

A. 2
B. 3
C. 4
D. 6

20. An integer $a \neq 0$ is called a divisor of an integer b if there exists an integer c such that

A. $ab = c$
B. $b = ac$
C. $a + b = c$
D. $a = bc$

SEPTEMBER 2018
MAT 201S
INTRODUCTION TO ABSTRACT ALGEBRA
1 HOUR 20 MINUTES

Candidate's Index Number
IE/MAT/KCS/18/
Signature:

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
SCHOOL OF EDUCATIONAL DEVELOPMENT AND OUTREACH
INSTITUTE OF EDUCATION

FIVE-SEMESTER BACHELOR OF EDUCATION (SANDWICH) PROGRAMME
FIRST YEAR, END-OF-FIRST SEMESTER EXAMINATION, SEPTEMBER 2018

SEPTEMBER 7, 2018 INTRODUCTION TO ABSTRACT ALGEBRA 9:40 AM–11:00 AM

SECTION B

Answer any three questions from this section.

1. Let A, B and C be sets. Prove that
(a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
(b) $(A - B) - C \subseteq A - (B - C)$

2. Assume that $\propto : S \to T$ and $\beta : T \to U$ are two mappings.
(a) If $\propto$ and β are onto, prove that the product $\beta \propto$ is also onto.
(b) If $\propto$ and β are one-to-one, prove that $\beta \propto$ is also one-to-one.

3. (a) Prove by the principle of mathematical induction that

$$1 \times 2 + 2 \times 3 + 3 \times 4 + \cdots + n \times (n + 1) = \frac{1}{3}n(n + 1)(n + 2)$$

for all $n \in \mathbb{N}$.

(b) Let $a, b \in \mathbb{Z}$. Show that $(-a).b = -(a.b)$.

4. (a) Show that the operation $a * b = 1 + ab$ defined on the set of integers $\mathbb{Z}$ has no identity element.
(b) Let $B = \mathbb{R} - \{0\}$. Define the relation $\mathcal{R}$ on B by $a\mathcal{R}b$ if and only if $ab > 0$. Show that $\mathcal{R}$ is an equivalence relation on B.

INSTITUTE OF EDUCATION

MAT 201S: INTRODUCTION TO ABSTRACT ALGEBRA

END OF SEMESTER EXAMINATION MARKING SCHEME (2018)

SECTION A

Each question is worth 2marks.

1. D
2. D
3. FALSE
4. FALSE
5. C
6. B
7. C
8. C
9. C
10. B
11. D
12. D
13. B
14. A
15. D
16. A
17. B
18. A
19. C
20. B

SECTION B

Each question is worth 20 marks.

1. (a) Let $x \in A \cap (B \cup C)$. ...... [M1/2]

Then

$x \in A$ and $x \in B \cup C$ [M1]

$\Rightarrow x \in A$ and $x \in B$ or $x \in C$ [M1/2] [illegible] $x \in A$ an[illegible] B or $x \in A$ and $x \in C$ [M1] m½ m½

When [illegible]

[illegible] $\in A$ and $x \in B$ then $x \in A \cap B$ so $x \in (A \cap B) \cup (A \cap C)$ [M1/2]

Also when

[illegible] A and [illegible] C then $x \in A \cap C$ so $x \in (A \cap B) \cup (A \cap C)$ [M1/2]

Thus,

$A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$ [A1]

Also, let $x \in$ [illegible] $\cup (A \cap C)$. [M1/2]

Then

$x \in A \cap B$ or $x \in A \cap C$ [M1]

When $x \in A \cap B$ then $x \in A$ and $x \in B$ [M1/2]

$\Rightarrow x \in A$ and $x \in B \cup C$ [M1/2]

Also, when $x \in A \cap C$ then $x \in A$ and $x \in C$ [M1/2]

$\Rightarrow x \in A$ and $x \in B \cup C$ [M1]

Thus,

$x \in A \cap (B \cup C)$ [A1]

$\Rightarrow (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$ [A1]

Therefore

[illegible] $(B \cup C)$ [illegible] $\cup (A \cap C)$ [A1]

(b) Let $x \in (A$ [illegible] $B) \cup C$ [M1]

Then

$x \in A$ but that $x \notin B$ and $x \notin C$ [M1 M1 M1]

Since [illegible] and $x \notin$ [illegible] follows that [B1/2 B1/2]

[illegible] [M1]

$\Rightarrow x \in A - (B - C)$ [M1]

Therefore,

$$(A - B) - C \subseteq A - (B - C) \quad [A1]$$

2. (a) Suppose that α and β are onto. [B1]
   Let $u \in U$. [M1]
   Since β is onto, there is a $t \in T$ such that $\beta(t) = u$. [M1/2 M1]
   Now, since α is onto, there is an $s \in S$ such that $\alpha(s) = t$. [M1/2 M1]
   Thus, given $u \in U$ we can find an $s \in S$ such that

$$(\beta\alpha)(s) = \beta(\alpha(s)) \quad [M1\ M1]$$
$$= \beta(t) \quad [M1]$$
$$= u \quad [M1]$$

Hence, $\beta\alpha$ is onto. [A1]

(b) Suppose that α and β are one-to-one. [B1]

Let $s_1, s_2 \in S$ such that

$$(\beta\alpha)(s_1) = (\beta\alpha)(s_2). \quad [M1\ M1]$$

Then

$$\beta(\alpha(s_1)) = \beta(\alpha(s_2))$$
$$\Rightarrow \alpha(s_1) = \alpha(s_2), \quad \text{since } \beta \text{ is one-to-one [M2 B1]}$$
$$\Rightarrow s_1 = s_2, \quad \text{since } \alpha \text{ is one-to-one [M2 B1]}$$

Therefore, $\beta\alpha$ is one-to-one. [A1]

3. (a) Let $P_n: 1 \times 2 + 2 \times 3 + 3 \times 4 + \cdots + n \times (n+1) = \frac{1}{3}n(n+1)(n+2)$

for all $n \in \mathbb{N}$. [B1]

For $n = 1$,

LHS of P_1: $1(1+1) = 2$ [M1]

RHS of P_1: $\frac{1}{3}(1)(1+1)(1+2) = \frac{6}{3} = 2$ [M1]

Thus, P_1 is true. [A1]

Assume that

$P_k: 1 \times 2 + 2 \times 3 + 3 \times 4 + \cdots + k \times (k+1) = \frac{1}{3}k(k+1)(k+2)$

is true. [M1]

Then,

$$1 \times 2 + 2 \times 3 + 3 \times 4 + \cdots + k \times (k+1) + (k+1)(k+2) \text{ -- [M1]}$$

$$= \frac{1}{3}k(k+1)(k+2) + (k+1)(k+2) \text{ -- [M1]}$$

$$= (k+1)(k+2)\left[\frac{1}{3}k + 1\right] \text{ [M1]}$$

$$= \frac{1}{3}(k+1)(k+2)(k+3) \text{ [M1]}$$

$$= P_{k+1} \text{ [M1]}$$

Thus, P_{k+1} is true. [M1]

Therefore, the statement P_n is true for all $n \geq 1$. [A1]

(b) Let $a \leftrightarrow [s, m]$ and $b \leftrightarrow [t, n]$. [M1/2 M1/2]

Then

$$-a \leftrightarrow [m, s] \text{ [M1]}$$

Thus,

$$(-a).b \leftrightarrow [m.s].[t.n] = [(m.t + s.n), (m.n + s.t)] \quad \cdots (1)$$ [M1 M1]

Also,

$$a.b \leftrightarrow [s, m].[t, n] = [(s.t + m.n), (s.n + m.t)]$$ [M1 M1] m½ m½

any term cor…

$$\Rightarrow -(a.b) \leftrightarrow [(s.n + m.t), (s.t + m.n)] \quad \cdots (2)$$ [M1]

f

From (1) and (2) we have that

$(-a).b = -(a.b)$ [A1]

4. (a) Suppose e is the identity element for $*$. [M1]

Then

[illegible] [M1]

[illegible] [M1]

[illegible] [M1]

[illegible] [M1]

UNIVERSITY OF CAPE COAST

INSTITUTE OF EDUCATION

[illegible]: INTRODUCTION TO ABSTRACT ALGEBRA

TUTORIAL 1

1. Let A, B, C be sets such that $A \subseteq C$ and $B \subseteq C$. Show that
   (a) $A \cup B \subseteq C$
   (b) $A \cup B = B \cup A$

2. Let A and B be subsets of a set S. Prove that $A \cap B = B \cap A$.

3. Determine whether $\propto: \mathbb{N} \longrightarrow \mathbb{N}$ defined by

   $$\propto(n) = -n \text{ For all } n \in \mathbb{N}$$

   is a mapping. Justify your answer.

4. Determine whether the mapping $\propto: \mathbb{R} \longrightarrow \mathbb{R}$ defined by
   $\propto(x) = 3 - 4x$ one-to-one.

5. Assume that $\propto: S \rightarrow T$ and $\beta: T \rightarrow U$ are two mappings.
   (a) If $\propto$ and β are onto, prove that the product $\beta \propto$ is also onto.
   (b) If $\propto$ and β are one-to-one, prove that $\beta \propto$ is also one-to-one.

6. Let $P(S)$ denote the set of all subsets of the non-empty set S, that is $P(s) = \{A: A \subseteq S\}$. Define the relation $\mathcal{R}$ on $P(s)$ by $A\mathcal{R}B$ if $A \subseteq B$.

   Determine if $\mathcal{R}$ is

   (a) Reflexive
   (b) Symmetric
   (c) Transitive.

7. A relation $\mathcal{R}$ on [illegible] is defined by $a\mathcal{R}b$ if $a^2 + b^2$ is even. Prove that $\mathcal{R}$ is an equivalence relation.