JUNE 2023
MAT 203SW
FURTHER CALCULUS
1 HOUR 20 MINUTES

Candidate's Index Number
Signature:

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
SCHOOL OF EDUCATIONAL DEVELOPMENT AND OUTREACH
INSTITUTE OF EDUCATION

FIVE-SEMESTER BACHELOR OF EDUCATION (SANDWICH) PROGRAMME
LEVEL 350, END-OF-FIRST SEMESTER EXAMINATIONS, JUNE 2023

20TH JUNE 2023 FURTHER CALCULUS 4:40 PM - 6:00 PM

SECTION B
(40 MARKS)

Answer any TWO questions from this Section.

1.

a. An oil storage tank ruptures at time $t = 0$ and oil leaks from the tank at a rate of $r(t) = 100e^{-0.01t}$ liters per minute. How much oil leaks out during the first hour? **(12 marks)**

b. Find the first derivative of the function $f(x) = \tanh^{-1}(\cos x)$. **(8 marks)**

2.

a. Evaluate the integral $\int x \cos x \, dx$. **(10 marks)**

b. Find the value of first derivative of $xe^y = x - y$ at the point $(1, 0)$. **(10 marks)**

3.

a. Find the absolute minimum value of the function $f(x) = x + \frac{1}{x}$ on the interval $[0.2,4]$. **(10 marks)**

b. Prove the reduction formula

$$\int (\ln x)^n dx = x(\ln x)^n - n \int (\ln x)^{n-1} dx$$

and use it to evaluate $\int (\ln x)^1 dx$. **(10 marks)**

4.

a. The curve with equation $y = \frac{1}{1+x^2}$ is called **a witch of Maria Agnesi**. Find an equation of the tangent line to this curve at the point $\left(-1, \frac{1}{2}\right)$. **(10 marks)**

b. Find the volume of the solid obtained by rotating about the x −axis the region under the curve $y = x^2$ from 0 to 1. **(10 marks)**