

JUNE 2023
 MAT 203SW
 FURTHER CALCULUS
 1 HOUR 20 MINUTES

sin
-cos
sin
-cos

Candidate's Index Number
Signature:

$xe^y = x - y$
 $xe^y dy + e^y dx = 1 - dy$
 $xe^y dy + e^y dx = 1 - dy$
 $xe^y dy + dy = 1 - e^y dx$

UNIVERSITY OF CAPE COAST
 COLLEGE OF EDUCATION STUDIES
 SCHOOL OF EDUCATIONAL DEVELOPMENT AND OUTREACH
 INSTITUTE OF EDUCATION

FIVE-SEMESTER BACHELOR OF EDUCATION (SANDWICH) PROGRAMME
 LEVEL 350, END-OF-FIRST SEMESTER EXAMINATIONS, JUNE 2023

20TH JUNE 2023

FURTHER CALCULUS

4:40 PM - 6:00 PM

SECTION B
 (40 MARKS)

4511:8836

Answer any TWO questions from this Section.

1.

- a. An oil storage tank ruptures at time $t = 0$ and oil leaks from the tank at a rate of $r(t) = 100e^{-0.01t}$ liters per minute. How much oil leaks out during the first hour? (12 marks)
- b. Find the first derivative of the function $f(x) = \tanh^{-1}(\cos x)$. (8 marks)

2.

- a. Evaluate the integral $\int x \cos x \, dx$. (10 marks)
- b. Find the value of first derivative of $xe^y = x - y$ at the point $(1, 0)$. (10 marks)

3.

- a. Find the absolute minimum value of the function $f(x) = x + \frac{1}{x}$ on the interval $[0.2, 4]$. (10 marks)

- b. Prove the reduction formula

$$\int (\ln x)^n dx = x(\ln x)^n - n \int (\ln x)^{n-1} dx$$

and use it to evaluate $\int (\ln x)^1 dx$.

(10 marks)

10000

200y
2x

4.

a. The curve with equation $y = \frac{1}{1+x^2}$ is called a **witch of Maria Agnesi**. Find an equation of the tangent line to this curve at the point $(-1, \frac{1}{2})$. (10 marks)

b. Find the volume of the solid obtained by rotating about the x -axis the region under the curve $y = x^2$ from 0 to 1. (10 marks)

Handwritten work for part (a):

$$y = \frac{1}{1+x^2}$$

$$\frac{dy}{dx} = \frac{0 \cdot (1+x^2) - 1 \cdot 2x}{(1+x^2)^2} = \frac{-2x}{(1+x^2)^2}$$

$$\left. \frac{dy}{dx} \right|_{(-1, \frac{1}{2})} = \frac{-2(-1)}{(1+1)^2} = \frac{2}{4} = \frac{1}{2}$$

$$y - \frac{1}{2} = \frac{1}{2}(x + 1)$$

$$y = \frac{1}{2}x + \frac{3}{4}$$

Handwritten work for part (b):

$$V = \pi \int_0^1 (x^2)^2 dx = \pi \int_0^1 x^4 dx = \pi \left[\frac{x^5}{5} \right]_0^1 = \frac{\pi}{5}$$

Handwritten work for a differential equation problem:

$$\frac{dy}{dx} = 1 - e^y$$

$$\frac{dy}{1 - e^y} = dx$$

$$\int \frac{dy}{1 - e^y} = \int dx$$

$$\int \frac{dy}{1 - e^y} = \int \frac{e^y dy}{e^y(1 - e^y)}$$

$$= \int \frac{e^y dy}{e^y - e^{2y}} = \int \frac{e^y dy}{e^y(1 - e^y)}$$

$$= \int \frac{dy}{1 - e^y}$$

$$= \int \frac{dy}{1 - e^y} = \int \frac{dy}{1 - e^y}$$