SECTION B

September 2019

1. a. Find the intervals on which $f(x) = x^3 - 3x^2 - 9x + 4$ is increasing or decreasing.

   b. First make a substitution and then use integration by parts to evaluate the integral $\int \theta^3 \cos(\theta^2) d\theta$.

2. a. The curve $y = \dfrac{1}{1+x^2}$ is called a **witch of Maria Agnesi**. Find an equation of the tangent line to this curve at the point $(-1, \tfrac{1}{2})$.

   b. Prove the reduction formula $\int (\ln x)^n dx = x(\ln x)^n - n\int (\ln x)^{n-1} dx$ and use it to evaluate $\int (\ln x)^2 dx$.

3. a. A bacteria population starts with 400 bacteria and grows at a rate of $r(t) = 450e^{1.5t}$ bacteria per hour. How many bacteria will there be after three hours?

   b. Use linear approximation to estimate the numbers

   i. $\sqrt{100.5}$

   ii. $e^{0.1}$

4. a. Evaluate $\int \dfrac{x+5}{x^2+x-2} dx$

   b. Suppose $f(\frac{\pi}{3}) = 4$ and $f'(\frac{\pi}{3}) = -2$, find $g'(\frac{\pi}{3})$ if $g(x) = \dfrac{\cos x}{f(x)}$.

   c. Evaluate $\int_a^b c\,dx$, where c is a constant.

April 2021

1. a. An oil storage tank ruptures at time $t = 0$ and oil leaks from the tank at a rate of $r(t) = 100e^{-0.01t}$ liters per minute. How much oil leaks out during the first hour?

   b. Find the absolute maximum and absolute minimum values of $f(x) = x - 2\arctan x$ on the interval $[0,4]$.

2. a. The curve with equation $x^2 + y^2 = 5x^4$ is called a **kampyle of Eudoxus**. Find an equation of the tangent line to this curve at the point $(1, 2)$.

   b. Find the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$.

3. a. Show that $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$ where $a \neq 0$.

   b. Suppose $f\left(\frac{\pi}{3}\right) = 4$ and $f'\left(\frac{\pi}{3}\right) = -2$, and let $g(x) = \frac{\cos x}{f(x)}$ and $h(x) = f(x)\sin x$. Find

i. $g'\left(\frac{\pi}{3}\right)$

ii. $h'\left(\frac{\pi}{3}\right)$

4. a. Find the linearization of the function $f(x) = \sqrt{x+3}$ at $a = 1$ and use it to approximate the numbers $\sqrt{3.98}$ and $\sqrt{4.05}$.

   b. If $f(x)$ and $g(x)$ are differentiable functions, prove the formula for integration by parts

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx$$

   and use that idea to evaluate $\int_1^2 \omega^2 \ln \omega \, d\omega$

December 2021

5. a. The equation of motion of a particle is $S(t) = 2t^3 - 5t^2 + 3t + 4$, where S is measured in centimeters and t in seconds.

   i. Find the acceleration as a function of time.

   ii. What is the acceleration after 2 seconds?

   b. Evaluate the integral $\int x^2\sqrt{1-x^3}\, dx$

6. a. If C is constant and f is differentiable function, then, show that

$$\frac{d}{dx}[Cf(x)] = Cf'(x)$$

   b. Find the volume of the solid obtained by rotating about the x –axis the region under the curve $y = \sqrt{x}$ from 0 to 1.

7. a. Find the first derivative of the function $f(x) = \tanh^{-1}(\sin x)$

   b. Prove the reduction formula

$$\int x^n e^x\, dx = x^n e^x - n\int x^{n-1} e^x\, dx$$

   and use it to evaluate $\int xe^x\, dx$.

8. a. Find $\left.\frac{dy}{dx}\right|_{(3,3)}$ if $x^3 + y^3 = 6xy$.

   b. A model for the basal metabolism rate, in kcal/h, of a young man is $R(t) = 85 - 0.18\cos(\pi t/12)$, where t is the time in hours measured from 5:00 am. What is the total basal metabolism of this man, $\int_0^{24} R(t)dt$, over a 24-hour time period?

9. a. Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ liters per minute, where $0 \le t \le 50$. Find the amount of water that flows from the tank during the first 10 minutes.

b. Find the critical number(s) of the function $h(p) = \frac{p-1}{p^2-4}$.

April 2022

1. a. An oil storage tank ruptures at time $t = 0$ and oil leaks from the tank at a rate of $r(t) = 100e^{-0.01t}$ liters per minute. How much oil leaks out during the first hour?

   b. Find the first derivative of the function $f(x) = \tanh^{-1}(\cos x)$

2. a. Evaluate the integral $\int x \cos x \, dx$

   b. Find $\left.\frac{dy}{dx}\right|_{(1,0)}$ if $xe^y = x - y$.

3. a. Find the absolute minimum value of the function $f(x) = x + \frac{1}{x}$ on the interval $[0.2,4]$.

   b. Prove the reduction formula $\int (\ln x)^n dx = x(\ln x)^n - n\int (\ln x)^{n-1} dx$ and use it to evaluate $\int (\ln x)^1 dx$.

4. a. The curve with equation $y = \frac{1}{1+x^2}$ is called **a witch of Maria Agnesi.** Find an equation of the tangent line to this curve at the point $\left(-1, \frac{1}{2}\right)$.

   b. Find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = x^2$ from 0 to 1.