JUNE 2023
PHY 301SW
CLASSICAL MECHANICS
2 HOURS

Candidate's Index Number
Signature:

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
SCHOOL OF EDUCATIONAL DEVELOPMENT AND OUTREACH
INSTITUTE OF EDUCATION

FIVE-SEMESTER BACHELOR OF EDUCATION (SANDWICH) PROGRAMME
LEVEL 350, END-OF-FIRST SEMESTER EXAMINATIONS, JUNE 2023

22ND JUNE 2023 **CLASSICAL MECHANICS** **7:00 AM - 9:00 AM**

(100 MARKS)

Answer THREE questions in all; Question 1 (40 marks) and any TWO others (30 marks each).

1.

a. If $\mathbf{A} = A_1\mathbf{i} - A_2\mathbf{j} + A_3\mathbf{k}$, show that $A = = \sqrt{A.A} = \sqrt{A_1^2 + A_2^2 + \sqrt{A_3^2}}$. **[8 marks]**

b. $\mathbf{A} = \mathbf{i} + \mathbf{j}$, $\mathbf{B} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\mathbf{C} = 4\mathbf{j} - 3\mathbf{k}$. Find (i) (A x B) x C, (ii) A x (B x C). **[8marks]**

c. Two particles have position vectors given by $\boldsymbol{r_1} = 2t\,\boldsymbol{i} + (3t^2 - 4t)\boldsymbol{k}$ and $\mathbf{r}_2 = (5\,t^2 - 12\,t + 4)\boldsymbol{i} + \mathrm{t}^3\boldsymbol{j} - 3\mathrm{t}\,\boldsymbol{k}$. Find (i) the relative velocity and (ii) the relative acceleration of the second $\boldsymbol{F}$ = particle with respect to the first at the instant where t = 2. **[8marks]**

d. A constant force F acting on a particle of mass m changes the velocity from $\boldsymbol{v_1}$ to $\boldsymbol{v_2}$ n time τ (i) Prove that $\mathbf{F} = m\left(\frac{\mathbf{v_2} - \mathbf{v_1}}{\tau}\right)$ (ii) Does the result in (i) holds if the force is variable? Explain. **[8marks]**

e. Show that the force field F defined by
$\mathbf{F} = (y^2z^3 - 6xz^2)\boldsymbol{i} + 2xyz^3\,\boldsymbol{j} + (3xy^2z^2 - 6x^2z)\boldsymbol{k}$ is a conservative force field. **[8marks]**

2. Suppose that the force acting on a system of particles are derivable from potential function V. Suppose that the system is conservative. Prove that if L = T – V is the Lagrangian function then $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_\alpha}\right) - \frac{\partial L}{\partial q_\alpha} = 0$ **[30 marks]**

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3. If the Hamiltonian $\mathcal{H} = \sum p_\alpha \dot{q}_\alpha - L$ where the summation extends from $\propto = 1$ to n, is expressed as a function of the coordinates q_α and momenta p_α, prove the Hamiltonian equation $\dot{p}_\alpha = -\frac{\partial \mathcal{H}}{\partial q_\alpha}$, $\dot{q}_\alpha = \frac{\partial \mathcal{H}}{\partial p_\alpha}$ regardless of whether $\mathcal{H}$ (i) does not or (b) does contain the variable time explicitly. **[30 marks]**

4. A particle moves in the xy plane under the influence of a central force depending only on its distance from the origin.

   a. Set up the Hamiltonian for the system.

   b. Write Hamilton's equation of motion. **[30marks]**

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