AUGUST 2020
MAT 301SW
ADVANCED CALCULUS I
2 HOURS

Candidate's Index Number
Signature:

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
SCHOOL OF EDUCATIONAL DEVELOPMENT AND OUTREACH
INSTITUTE OF EDUCATION

FIVE-SEMESTER BACHELOR OF EDUCATION (SANDWICH) PROGRAMME
SECOND YEAR, SECOND SEMESTER EXAMINATION, AUGUST 2020

AUGUST 29, 2020 ADVANCED CALCULUS I 2:00 PM – 2:40 PM

This paper consists of two sections, A and B. Answer ALL questions in Section A and TWO questions in Section B. Section A will be collected after the first 40 minutes.

SECTION A

Answer ALL questions in this section.

Items 1 to 20 are stems followed by four options lettered A to D. Read each item carefully and circle the letter of the correct or best option.

1. Find the domain of the function $f(x, y) = y^2 \ln x$.
   A. $\{(x, y): y > 0\}$
   B. $\{(x, y): x > 0\}$
   C. $\{(x, y): x < 0\}$
   D. $\{(x, y): y < 0\}$

2. Evaluate $\lim_{(x,y)\to(1,0)} \frac{y}{x+y-1}$ along the path $x = 1$.
   A. Does not exist
   B. $\frac{1}{2}$
   C. 1
   D. 3

3. Determine the set of points at which the function $f(x, y) = \frac{x-y}{x^2+y^2}$ is continuous.
   A. $\{(x, y) \in \mathbb{R}^2: (x, y) \neq (0, 0)\}$
   B. $\{(x, y) \in \mathbb{R}^2: (x, y) \neq (1, 0)\}$
   C. $\{(x, y) \in \mathbb{R}^2: (x, y) \neq (1, -1)\}$
   D. $\{(x, y) \in \mathbb{R}^2\}$

4. Suppose that $f(1, 0) = 3$. If $f(x, y)$ is continuous, find $\lim_{(x,y)\to(1,0)} f(x, y)$.
   A. Does not exist
   B. 3
   C. 4
   D. 8

5. If $f(x,y) = \sin(xy)$, find $f_x(x,y)$.
A. $y\cos(xy)$
B. $x\cos(xy)$
C. $-y\cos(xy)$
D. $-x\cos(xy)$

6. If $f(x,y) = e^{xy^3}\ln(x^2)$, find f_y.
A. $3y^2xe^{xy^3} + 2x^{-1}$
B. $3y^2xe^{xy^3}\ln(x^2)$
C. $3y^2xe^{xy^3} + 2x^{-1}e^{xy^3}$
D. $2x^{-1}$

7. Find the second partial derivative f_{yx} of $f(x,y) = y^{10} - x^2y$.
A. $90y^8 - 2xy$
B. $10y^9 - 2y$
C. $-2y$
D. $-2x$

8. If $z = f(x,y) = xe^{y+1}$, find the differential dz.
A. $xdx + e^{y+1}dy$
B. $xdx + xe^{y+1}dy$
C. $e^{y+1}dx + xe^{y+1}dy$
D. $xe^{y+1}dx + e^{y+1}dy$

9. Find the chain rule for $\frac{dw}{dt}$ if $w = f(x,y,z)$ with $x = g_1(t)$, $y = g_2(t)$ and $z = g_3(t)$.
A. $\frac{\partial w}{\partial t} = \frac{\partial z}{\partial x}\frac{dx}{dy} + \frac{\partial z}{\partial y}\frac{dy}{dx} + \frac{\partial z}{\partial z}\frac{dz}{dt}$
B. $\frac{\partial w}{\partial t} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} + \frac{\partial z}{\partial z}\frac{dz}{dt}$
C. $\frac{\partial w}{\partial t} = \frac{\partial z}{\partial x}\frac{dy}{dt} + \frac{\partial z}{\partial y}\frac{dz}{dt} + \frac{\partial z}{\partial z}\frac{dx}{dt}$
D. $\frac{\partial w}{\partial t} = \frac{\partial z}{\partial x}\frac{dt}{dt} + \frac{\partial z}{\partial y}\frac{dt}{dt} + \frac{\partial z}{\partial z}\frac{dt}{dt}$

10. Find $\frac{dy}{dx}$ if $x^3 + y^3 = 12xy$.
A. $-\frac{x^2-4y}{y^2-4x}$
B. $-\frac{x^2-2y}{y^2-2x}$
C. $-\frac{x^2+4y}{y^2+4x}$
D. $-\frac{x^2+2y}{y^2+2x}$

11. Evaluate the double integral $\iint_R xdA$, where $R = \{(x,y): 0 \le x \le 2, 1 \le y \le 2\}$.
A. 6
B. 4
C. 2
D. -2

12. Evaluate $\iint_D 4dA$, where $D = \{(x,y): 0 \le x \le 1, 2x^3 \le y \le 1 + x^3\}$.
   A. 0
   B. 2
   C. 3
   D. 4

13. Evaluate $\iint_D 4dA$, where $D = \{(x,y): 0 \le y \le 1,\ 0 \le x \le y^3\}$.
   A. 0
   B. 1
   C. 2
   D. 4

14. If $f(x,y) = 1$, evaluate $\iint_R f(x,y)\,dA$, where $R = \{(r,\theta) : 1 \le r \le 2, 0 \le \theta \le \pi\}$.
   A. 2π
   B. $\frac{3\pi}{2}$
   C. π
   D. $\frac{\pi}{2}$

15. Express $\int_{-a}^{a}\int_{0}^{\sqrt{a^2-x^2}} (x^2+y^2)^{3/2}\,dy\,dx$ in polar coordinates.
   A. $\int_{-\pi}^{\pi}\int_0^a r^4\,dr\,d\theta$
   B. $\int_0^{\pi/2}\int_0^a r^4\,dr\,d\theta$
   C. $\int_0^{\pi}\int_0^a r^3\,dr\,d\theta$
   D. $\int_0^{\pi}\int_0^a r^4\,dr\,d\theta$

16. Express $\iiint_E z\,dV$, where $E = \{(x,y,z): 0 \le x \le 1, 0 \le y \le 1-x, 0 \le z \le 1-x-y\}$ in a form that can be evaluated.
   A. $\int_0^{1-x-y}\int_0^{1-x}\int_0^1 z\,dx\,dy\,dz$
   B. $\int_0^1\int_0^{1-x}\int_0^{1-x-y} z\,dz\,dy\,dx$
   C. $\int_0^1\int_0^{1-x-y}\int_0^{1-x} z\,dy\,dz\,dx$
   D. $\int_0^{1-x}\int_0^1\int_0^{1-x-y} z\,dz\,dx\,dy$

17. Let $E = \{(r,\theta,z) : 0 \le \theta \le 2\pi, 0 \le r \le 1, 1-r^2 \le z \le 4\}$ and $f(x,y,z) = \sqrt{x^2+y^2}$
   Express $\iiint_E f(x,y,z)\,dV$ in cylindrical coordinates.
   A. $\int_0^{2\pi}\int_0^1\int_{1-r^2}^4 r^2\,dz\,dr\,d\theta$
   B. $\int_0^{2\pi}\int_0^1\int_{1-r^2}^4 r^2\,dr\,dz\,d\theta$
   C. $\int_0^{2\pi}\int_0^1\int_{1-r^2}^4 r^2\,d\theta\,dr\,dz$
   D. $\int_0^{2\pi}\int_0^1\int_{1-r^2}^4 r^2\,dz\,d\theta\,dr$

18. If E is a spherical wedge given by $E = \{(\rho,\theta,\phi) : 0 \le \rho \le 1, 0 \le \theta \le 2\pi, 0 \le \phi \le \pi\}$, express $\iiint_E e^{(x^2+y^2+z^2)^{3/2}} dV$ in spherical coordinates.

A. $\int_0^{\pi}\int_0^{2\pi}\int_0^1 e^{\rho^{3/2}}\rho^2 \sin\phi \; d\rho\, d\theta\, d\phi$

B. $\int_0^{\pi}\int_0^{2\pi}\int_0^1 e^{\rho^{3/2}}\rho \sin\phi \; d\rho\, d\theta\, d\phi$

C. $\int_0^{\pi}\int_0^{2\pi}\int_0^1 e^{\rho^3}\rho^2 \sin\phi \; d\rho\, d\theta\, d\phi$

D. $\int_0^{\pi}\int_0^{2\pi}\int_0^1 e^{\rho^3}\rho \sin\phi \; d\rho\, d\theta\, d\phi$

19. Let $f(x,y) = -x^2 - y^2 - 2x - 6y + 14$. Find the critical point of f.

A. $(1,3)$

B. $(3,1)$

C. $(-3,-1)$

D. $(-1,-3)$

20. Suppose that f is continuous on a disk with center (a,b), such that $f_x(a,b) = 0$ and $f_y(a,b) = 0$. If $D(a,b) > 0$ and $f_{xx}(a,b) < 0$, then which of the following is true about $f(a,b)$?

A. Absolute maximum.

B. Local maximum.

C. Local minimum.

D. Saddle.