SEPTEMBER 2019
MAT 301SW
ADVANCED CALCULUS II
2 HOURS

Candidate's Index Number
Signature:

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
SCHOOL OF EDUCATIONAL DEVELOPMENT AND OUTREACH
INSTITUTE OF EDUCATION

FIVE-SEMESTER BACHELOR OF EDUCATION (SANDWICH) PROGRAMME
SECOND YEAR, END-OF-THIRD SEMESTER EXAMINATION, SEPTEMBER 2019

SEPTEMBER 3, 2019 ADVANCED CALCULUS II 2:00 PM - 2:40 PM

This paper consists of two sections, A and B. Answer all the questions in Section A and THREE questions in Section B. Section A will be collected after the first 40 minutes.

SECTION A
(20 MARKS)

Answer all the questions in this Section.

1. Find the domain of the vector function $r(t) = ti + \ln t\, j + \sqrt{t-2}k$.
   A. $[0,\infty)$
   B. $[2,\infty)$
   C. $[0,2]$
   D. $(0,2)$

2. Find $\lim_{t\to 0}\langle 2t, e^t, \frac{\sin t}{t}\rangle$.
   A. $\langle 1,0,1\rangle$
   B. $\langle 0,1,0\rangle$
   C. $\langle 0,0,0\rangle$
   D. $\langle 0,1,1\rangle$

3. Determine for what values of t the vector-valued function
   $r(t) = \langle \frac{2+t}{t-2}, e^t, \cos t\rangle$ is continuous.
   A. $(-\infty,2)\cup(2,\infty)$
   B. $(-\infty,1)\cup(1,\infty)$
   C. $(1,\infty)$
   D. $(-\infty,\infty)$

4. Find the derivative of the vector function $r(t) = \langle 1+2t,\ e^{t^2}, \cos t \rangle$

A. $\langle 2, 2te^{t^2}, -\sin t \rangle$

B. $\langle 2, 2t, \sin t \rangle$

C. $\langle 2, 2e^{t^2}, \sin t \rangle$

D. $\langle 2, t^2e^{t^2}, -\sin t \rangle$

5. Evaluate the indefinite integral $\int (\sin t\, i + 5t^4\, j + t^{-1} k)dt$.

A. $\cos t\, i + 20j + k + c$

B. $\cos t\, i + 20t^3 j - t^{-2} k + c$

C. $\cos t\, i + t^5 j + \ln t\, k + c$

D. $-\cos t\, i + t^5 j + \ln t\, k + c$

6. Evaluate $\int_0^1 \langle 2t, \sin \pi t \rangle dt$.

A. $\langle 1, -2/\pi \rangle$

B. $\langle 1, 2/\pi \rangle$

C. $\langle \pi, 1 \rangle$

D. $\langle \pi, 0 \rangle$

7. Find the tangent vector to the curve given by $r(t) = t^2\, i + tj$.

A. $t^2 + t$

B. $2t + 1$

C. $< 1, 2t >$

D. $< 2t, 1 >$

8. Find the unit tangent vector to the curve given by $r(t) = 3ti + 4tj$

A. $\langle 3, 5 \rangle$

B. $\langle 3/5, 4/5 \rangle$

C. $\langle 3t/5, 4t/5 \rangle$

D. $\langle 3t, 4t \rangle$

9. Find the directional derivative $D_u f(x,y)$ of $f(x,y) = x^3y^3$ in the direction of the unit vector $u = \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$.

A. $3\sqrt{3}x^2y^3 + 3x^3y^2$
B. $\frac{3\sqrt{3}}{2}x^3y^2 + \frac{3}{2}x^2y^3$
C. $\frac{\sqrt{3}}{2}x^3y^3 + \frac{1}{2}x^3y^3$
D. $\frac{3\sqrt{3}}{2}x^2y^3 + \frac{3}{2}x^3y^2$

10. Find $\nabla f(x,y)$ if $f(x,y) = e^x \sin y$.

A. $\langle e^x \sin y, e^x \cos y \rangle$
B. $\langle e^x, \cos y \rangle$
C. $\langle \sin y + xe^x, e^x \rangle$
D. $\langle \sin y + e^x, \cos y \rangle$

11. Evaluate $\int_C 2x\, ds$ where the parametric equations of C are

$x = 1, \quad y = t, \quad 1 \le t \le 2.$

A. $\sqrt{2}$
B. 2
C. -2
D. $-\sqrt{2}$

12. Find the value of k for which the integral $\int_C F.dr$ where

$F(x,y) = (kx^2y + 2)i + (x^3 + 4y^3)j$ is independent of path.

A. -2
B. 1
C. 2
D. 3

13. Find the parametric equations of the of the curve C, where C is the arc of the parabola $y = 1 + x$ from $(1,3)$ to $(2,4)$.

A. $x = t, \quad y = 1 + t, \quad 1 \le t \le 4$
B. $x = t, \quad y = 1 + t, \quad 2 \le t \le 4$
C. $x = t, \quad y = 1 + t, \quad 1 \le t \le 2$
D. $x = t, \quad y = 1 + t, \quad 3 \le t \le 4$

14. Evaluate $\int_{(0,1,0)}^{(2,2,2)} F \cdot dr$ if the potential function for F is $f(x, y, z) = xy^2 + z + k$.

A. 10
B. 9
C. 8
D. 4

15. Evaluate $\oint_C (x^6)\, dx + (2x)\, dy$, where the region D enclosed by C is given by $D = \{(x, y): 0 \leq x \leq 1, 0 \leq y \leq 1\}$.

A. 2
B. 4
C. 6
D. 8

16. If $F(x, y, z) = xy\, i + yz\, j + xzk$, find curl F.

A. $2yxi + xj + yk$
B. $-yi - zj + xk$
C. $-yi - zj - xk$
D. $-2yxi - xj - yk$

17. Find div F if $F(x, y, z) = x\, i + y\, j + z\, k$.

A. 1
B. 2
C. 3
D. $3xyz$

18. Evaluate $\iint_S y\, dS$, where S is the surface $z = x + y, 0 \leq x \leq 1, 0 \leq y \leq 2$.

A. $3\sqrt{3}$
B. $2\sqrt{3}$
C. $\sqrt{3}$
D. 2

If $F(x,y,z) = P(x,y,z)i + Q(x,y,z)j + R(x,y,z)k$ then the surface integral $\iint_S F.dS$ is given by

A. $\iint_D \left(-P\frac{\partial z}{\partial x} - Q\frac{\partial z}{\partial y} + R\right) dA$

B. $\iint_D \left(-P\frac{\partial z}{\partial x} - Q\frac{\partial z}{\partial y} - R\right) dA$

C. $\iint_D \left(-Q\frac{\partial z}{\partial x} - P\frac{\partial z}{\partial y} + R\right) dA$

D. $\iint_D \left(-Q\frac{\partial z}{\partial x} - P\frac{\partial z}{\partial y} - R\right) dA$

). The Stoke's theorem states that the integral $\int_C F.dr$ is given by

A. $\iint_S \text{div } F.dS$

B. $\iint_S \text{curl } F.dS$

C. $\iint_S F.dS$

D. $\iint_S \nabla F.dS$

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SECTION B

Answer any three questions from this section.

1. (a) Find the derivative of $r(t) = \sin(t^2)\, i + e^{\cos t}\, j + t \ln t\, k$. [8 marks]

   (b) Evaluate $\int_C (2 + x^2 y)\, ds$, where C is the upper half of the unit circle $x^2 + y^2 = 1$. [12 marks]

2. (a) Find the directional derivative $D_u f(x, y, z)$ for $f(x, y, z) = \sin(yz) + \ln x^2$ in the direction of the vector $v = \langle 1, 1, -1 \rangle$. [8 marks]

   (b) Evaluate the integral $\int_0^2 \left(\frac{1}{t+1} i + e^{t-2} j + te^t k \right) dt$ [12 marks]

3. (a) Evaluate $\oint_C y^2\, dx + 3xy\, dy$, where the region D enclosed by C is given by $D = \{(r, \theta) : 1 \le r \le 2, 0 \le \theta \le \pi\}$. [8 marks]

   (b) Prove that the line integral

   $$\int_{(-1,2)}^{(3,1)} (y^2 + 2xy)\, dx + (x^2 + 2xy)\, dy$$

   is independent of path and find its value. [12 marks]

4. (a) If $F(x, y, z) = xz\, i + xyz\, j - y^2 k$, find curl F. [8 marks]

   (b) Use Stoke's Theorem to evaluate $\iint_S \text{curl } F \cdot dS$, where $F(x, y, z) = yz\, i + xz\, j + xy\, k$ where the vector equation of the boundary curve C is $r(t) = 2\cos t\, i + 2\sin t\, j + 5\, k$, $0 \le t \le 2\pi$. [12 marks]