MATHS / ECONS

APRIL 2023
MAT 302SW
ADVANCED CALCULUS II
30 MINUTES

Candidate's Index Number
IE/MAT/KBI/21/0110
Signature:

18

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
SCHOOL OF EDUCATIONAL DEVELOPMENT AND OUTREACH
INSTITUTE OF EDUCATION

FIVE-SEMESTER BACHELOR OF EDUCATION (SANDWICH) PROGRAMME
LEVEL 400, FIRST SEMESTER QUIZ 1, APRIL 2023

3RD APRIL 2023 ADVANCED CALCULUS II 3:30 PM - 4:00 PM

Answer ALL the questions.
(20 MARKS)

Items 1 to 20 are stems followed by four options lettered A to D. Read each item carefully and circle the letter of the correct or best option.

1. Find the domain of the vector function $r(t) = \langle \ln(4-t), t+1, \sqrt{t} \rangle$.
   A. $(-\infty, \infty)$
   B. $(-\infty, 4]$
   C. $[0, 4]$
   D. $[0, 4)$

   $0 \le t < 4$
   $[0, 4)$

2. Find $\lim_{t \to 0} \langle 3t^2, \cos t, 1 + \sin t \rangle$.
   A. $\langle 0, 1, 1 \rangle$
   B. $\langle 0, 0, 1 \rangle$
   C. $\langle 0, 1, 0 \rangle$
   D. $\langle 0, -1, -1 \rangle$

3. Determine for what values of t the vector-valued function $r(t) = \langle e^{2t}, \ln(t-1), \sin t \rangle$ is continuous.
   A. $(-\infty, 1)$
   B. $(-\infty, -1)$
   C. $(-1, \infty)$
   D. $(1, \infty)$

   $t-1>0$
   $t>1$

4. Find the derivative of the vector function $r(t) = \langle t^4, e^{2t}, \cos(4t) \rangle$.
   A. $\langle 4t^4, e^{2t}, -4\sin(4t) \rangle$
   B. $\langle 4t^3, 2e^{2t}, -4\sin(4t) \rangle$
   C. $\langle 4t^3, 2e^{2t}, 4\sin(4t) \rangle$
   D. $\langle 4t^3, 2e^{t}, 4\sin(4t) \rangle$

   $-\sin 4t$

   3

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5. Evaluate the indefinite integral $\int(\cos t\, i + 3t^2\, j + t^{-1} k)dt$.

A. $\sin t\, i + 6t j + k + c$
B. $\sin t\, i + 6t j - t^{-2} k + c$
C. $\sin t\, i + t^3 j + \ln t\, k + c$
D. $-\sin t\, i + 3t^3 j + \ln t\, k + c$

6. Evaluate $\int_0^1 (4t^3, \cos \pi t)dt$.

A. $\langle 1, 0\rangle$
B. $\langle 0, 1, \rangle$
C. $\langle \pi, 1\rangle$
D. $\langle \pi, 0\rangle$

7. Find the tangent vector to the curve given by $r(t) = 2t\, i + e^{2t+\tan t} j + \ln t\, k$.

A. $t\, i + (2 + \sec^2 t)\, e^{2t+\tan t} j + t^2\, k$
B. $t\, i + e^{2t+\tan t} j + t\, k$
C. $2\, i + (2 + \operatorname{cosec}^2 t)\, e^{2t+\tan t} j + 2t\, k$
D. $2\, i + (2 + \sec^2 t)\, e^{2t+\tan t} j + t^{-1}\, k$

8. Find the unit tangent vector to the curve given by $r(t) = \sin(2t)\, j + \cos(2t)\, k$.

A. $2\sin(2t)\, j - 2\cos(2t)\, k$
B. $\cos(2t)\, j - \sin(2t)\, k$
C. $\frac{1}{2}\sin(2t) j - \frac{1}{2}\cos(2t) k$
D. $\frac{1}{2}\cos(2t) j - \frac{1}{2}\sin(2t)\, k$

9. Determine the arc length for the curve $r(t) = \langle 2\cos t, 2\sin t\rangle$ on the interval $0 \le t \le 2\pi$.

A. 24π
B. 12π
C. 4π
D. 2π

$(2\sin t)^2, (-2\cos t)^2$

$4\sin^2 t + 4\sin^2 t$

$\sqrt{4} = 2$

$\int 2t$

10. Find the arc length function for $r(t) = \langle \cos(4t), \sin(4t)\rangle$.

A. $4t$
B. $6t$
C. $8t$
D. $12t$

11. Given that $r'(t) = t\, i + 2k$ and $r'(t) \times r''(t) = 4j$, find the curvature of $r(t)$.

A. $(t^2 + 4)^{\frac{3}{2}}$
B. $4(t^2 + 4)^{\frac{3}{2}}$
C. $(t^2 + 4)^{-\frac{3}{2}}$
D. $4(t^2 + 4)^{-\frac{3}{2}}$

12. Find the acceleration vector if the position of an object moving in the xy-plane is given by $r(t) = \langle t^2, e^{2t}\rangle$.

A. $\langle 2t, 2e^{2t}\rangle$
B. $\langle 2, 4e^{2t}\rangle$
C. $\langle \frac{1}{3}t^3, \frac{1}{3}e^{2t}\rangle$
D. $\langle \frac{1}{9}t^4, \frac{1}{9}e^{2t}\rangle$

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13. Find the directional derivative $D_u f(x,y)$ if $f(x,y) = x^4 + y^3$ and u is the unit vector given by the angle $\theta = \pi$.
   A. $4x^3 + 3y^2$
   B. $4x^3$
   C. $-4x^3$
   D. $-4x^3 - 3y^2$

$(3,0)$ $(0,4)$

$(0-3) + (4-0)$

14. Find the directional derivative $D_u f(x,y)$ for $f(x,y) = xy + y^2$ in the direction of the vector from $(3,0)$ to $(0,4)$.
   A. $-\frac{3}{5}y - \frac{4}{5}(x+2y)$
   B. $-\frac{3}{5}y + \frac{4}{5}(x+2y)$
   C. $-\frac{3}{5}y + \frac{4}{5}(x+2y)$
   D. $3y + 4(x+2y)$

$y + (x+2y)$

$\sqrt{(3-0)^2 + \ldots}$

$\sqrt{9+\ldots}$

$\sqrt{25} = 5$

15. Find $\nabla f(x,y)$ if $f(x,y) = xe^y$.
   A. $\langle e^y, xe^y \rangle$
   B. $\langle xe^y, e^y \rangle$
   C. $\langle x, e^y \rangle$
   D. $\langle e^y, x + e^y \rangle$

16. If $f(x,y,z) = z\cos(yx)$, find the gradient of f.
   A. $\langle \sin(yx), xz\cos(yx), xy\cos(yx) \rangle$
   B. $\langle \sin(yx), -xz\cos(yx), -xy\cos(yx) \rangle$
   C. $\langle -zy\sin(yx), -xz\sin(yx), \cos(yx) \rangle$
   D. $\langle zy\sin(yx), xz\sin(yx), \cos(yx) \rangle$

Use the following information to answer questions 17 to 19.

Let $f(x,y) = \sin(xy)$.

$y\cos(xy)$, $x\cos(xy)$

$(0,1)$

17. Find $\nabla f(1,0)$.
   A. $\langle 0,1 \rangle$
   B. $\langle 1,0 \rangle$
   C. $\langle 1,1 \rangle$
   D. $\langle 2,2 \rangle$

18. Find the maximum rate of change of f at the point $(1,0)$.
   A. 2π
   B. π
   C. 2
   D. 1

19. Find the direction in which the maximum rate of change of f at the point $(1,0)$ occurs.
   A. $\langle 0,-1 \rangle$
   B. $\langle 0,1 \rangle$
   C. $\langle 1,0 \rangle$
   D. $\langle 1,1 \rangle$

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20. Find the rate of change of $f = 5xy^2 - 4x^3y$ at $P(1,2)$ in the direction of the vector $u = \langle \frac{5}{13}, \frac{12}{13} \rangle$.

A. 3/13
B. 17/13
C. 172/13
D. 200/13

LIGHT BILL

7th September [Sept 47 ਭਾਦੋਂ]

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