JANUARY 2024
MAT 302SW
ADVANCED CALCULUS II
1 HOUR 20 MINUTES

Candidate's Index Number

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
SCHOOL OF EDUCATIONAL DEVELOPMENT AND OUTREACH
INSTITUTE OF EDUCATION

FIVE-SEMESTER BACHELOR OF EDUCATION (SANDWICH) PROGRAMME
LEVEL 400, END-OF-FIRST SEMESTER EXAMINATION, JANUARY 2024

8TH JANUARY 2024 **ADVANCED CALCULUS II** **9:40 AM - 11:00 AM**

SECTION B
[40 MARKS]

Answer any TWO questions from this section.
Please, note that if you answer more than three questions, only the first three will be marked.

1.

a. Find the derivative of $r(t) = at\cos(3t)\,\boldsymbol{i} + b\sin^3 t\,\boldsymbol{j} + c\cos^3 t\,\boldsymbol{k}$. [7 marks]

b. Evaluate $\int_C ydx + z\,dy + xdz$ where C is the line segment from $(2, 0, 0)$ to $(3, 4, 5)$. [13 marks]

2.

a. Let S be the portion of the plane $x + y + z = 1$ over $D = \{(x, y): 0 \le x \le 1, 0 \le y \le 1 - x\}$.
Use Stoke's Theorem to evaluate $\int_C F \,.\, dr$, where the surface S is the planar region enclosed by C. [12 marks]

b. Evaluate $\oint_C y^2\,dx + 3xy\,dy$, where the region D enclosed by C is given by $D = \{(r, \theta): 1 \le r \le 2, 0 \le \theta \le \pi\}$. [8 marks]

3.

a. Let $f(x,y) = x^2y^3 - 4y$. Find

i. $\nabla f(x,y)$ [4 Marks]

ii. the directional derivative of f in the direction of the vector $v = 2i + 5j$ using your answer in part (i). [8 marks]

b. Evaluate the integral $\int_0^1 \left(\frac{4}{1+t^2}j + \frac{2t}{1+t^2}k\right) dt$. [8 marks]

4.

a. Evaluate $\oint_C y^2\,dx + 3xy\,dy$, where the region D enclosed by C is given by $D = \{(r,\theta): 1 \le r \le 2, 0 \le \theta \le \pi\}$. [8 marks]

b. Prove that the line integral

$$\int_{(-1,2)}^{(3,1)} (y^2 + 2xy)dx + (x^2 + 2xy)dy$$

is independent of path and find its value. [12 marks]

$2\ln(1+t^2)^m \quad 6t^{-1}\ln$

$\frac{2}{1+t^2} \quad 2\left(\frac{1}{1+t^2}\right.$

$4+$

$2t+2$

$4(1+t^2)$

$4(1+t^2)^{-1}$

$-4(1+t^2)^{-2}\cdot 2t$

$-6t(1+t^2)^{-2}$

$\ln(1+t^2)$

$\frac{1}{1+t^2}\cdot 2t = t^2 + 2\ln(1+t^2)$

$2t$

$\frac{1}{1+t^2}\cdot 6t = \frac{2t}{1+t^2}$

$\frac{-6t}{(1+t^2)^2} = \frac{-6t}{(1+t^2)(1+t^2)}$

$= \frac{-6t}{1+t^2+t^2+t^4}$

$\frac{-6t}{t^4+2t^2+1}$