

OCTOBER 2023
MAT 301SW
ADVANCED CALCULUS I
30 MINUTES

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MATHS / SCIENCE
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UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
SCHOOL OF EDUCATIONAL DEVELOPMENT AND OUTREACH
INSTITUTE OF EDUCATION

FIVE-SEMESTER BACHELOR OF EDUCATION (SANDWICH) PROGRAMME
LEVEL 350, SECOND SEMESTER QUIZ, SEPTEMBER/OCTOBER 2023

2ND OCTOBER 2023

ADVANCED CALCULUS I

11:00 AM - 11:30 AM

Answer ALL the questions.
(20 MARKS)

Items 1 to 20 are stems followed by four options lettered A to D. Read each item carefully and circle the letter of the correct or best option.

- Find the domain of the function $f(x, y) = \frac{3x}{y-x}$.
A. $\{(x, y): x \neq 1\}$
B. $\{(x, y): x \geq 3\}$
 C. $\{(x, y): y \neq x\}$
D. $\{(x, y): y > 1\}$
- Describe the range of the function $f(x, y) = x^2 + y^2$.
A. $(0, 2)$
B. $[0, 2]$
C. $[0, \infty)$
 D. $(-\infty, \infty)$
- Evaluate $\lim_{(x, y) \rightarrow (4, 1)} f(x, y)$, where $f(x, y) = 4x$.
A. 4
B. 8
C. 9
 D. 16

4. Evaluate $\lim_{(x,y) \rightarrow (3,5)} f(x,y)$, where $f(x,y) = 7 - y$.

- A. -2
- B. 0
- C. 2
- D. 6

5. Calculate $\lim_{(x,y) \rightarrow (1,3)} \frac{6x-2y}{y+x^2}$.

- A. 0
- B. 2
- C. 3
- D. 4

6. Evaluate $\lim_{(x,y) \rightarrow (0,0)} \frac{2xy}{x^2+y^2}$ along the path $x = y$.

- A. 1
- B. $1/2$
- C. $1/7$
- D. 0

$$\frac{2xy}{x^2+y^2} = \frac{2y^2}{2y^2} = 1$$

7. Determine the set of points at which the function $f(x,y) = \frac{x^2+y^2}{x^2+y^2-18}$ is continuous?

- A. $\{(x,y) \in \mathbb{R}^2: (x,y) \neq (0,0)\}$
- B. $\{(x,y) \in \mathbb{R}^2: (x,y) \neq (\pm 1, \pm 1)\}$
- C. $\{(x,y) \in \mathbb{R}^2: (x,y) \neq (\pm 3, \pm 3)\}$
- D. $\{(x,y) \in \mathbb{R}^2: (x,y) \neq (18, -1)\}$

$$3^2 + 3^2 = 18$$

8. Suppose that $\lim_{(x,y) \rightarrow (3,1)} f(x,y) = 8$. If $f(x,y)$ is continuous at the point $(3,1)$, find $f(3,1)$.

- A. Undefined
- B. 8
- C. 4
- D. 3

9. If $f(x,y) = \sin(x^3y^2)$, find $f_x(x,y)$.

- A. $-\cos(x^3y^2)$
- B. $-3x^2 \cos(x^3y^2)$
- C. $\cos(x^3y^2)$
- D. $3x^2 \cos(x^3y^2)$

$$3x^2y^2 \cos(x^3y^2)$$

10. If $f(x,y) = e^{xy}$, find f_y .

- A. e^y
- B. xe^{xy}
- C. ye^{xy}
- D. xe^x

$$xe^{xy}$$

11. Find the second partial derivative f_{xy} of $f(x, y) = y^2 + xy$.

- A. $2y + 1$
- B. x
- C. y
- D. 1

$$f_x = y^2 + y$$

$$f_{xy} = 1$$

12. If $z = f(x, y) = y^2x$, find the differential dz .

- A. $2ydx + xdy$
- B. $y^2dx + 2yx dy$
- C. $xdx + 2xdy$
- D. $xy^2dx + xydy$

$$y^2dx + 2yx dy$$

13. If $z = xy$, where $x = 4t$ and $y = t^2$, find $\frac{dz}{dt}$.

- A. $4y + tx$
- B. $4x + 4t^2$
- C. $4 + 2t$
- D. $4y + 2xt$

$$\frac{\partial z}{\partial x} \frac{dx}{dt} + \frac{\partial z}{\partial y} \frac{dy}{dt}$$

$$y \cdot 4 + x \cdot 2t$$

$$4y + 2xt$$

14. Find the chain rule for $\frac{\partial w}{\partial t}$ if $w = f(x, y)$ with $x = h_1(t, s)$, and $y = h_2(t, s)$.

- A. $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial t}$
- B. $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial t} \frac{\partial t}{\partial x} + \frac{\partial w}{\partial t} \frac{\partial t}{\partial y}$
- C. $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \frac{\partial y}{\partial t} + \frac{\partial w}{\partial y} \frac{\partial x}{\partial t}$
- D. $\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial t}$

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial t}$$

15. Find $\frac{dy}{dx}$ if $x + y^2 = 5xy$. $x + y^2 - 5xy = 0$

- A. $-\frac{1-5y}{2y-5x}$
- B. $-\frac{1-5x}{1+2y}$
- C. $-\frac{1+2y}{5y+5x}$
- D. $-\frac{2x-1}{5y-2}$

$$\frac{f_x}{f_y} = -\frac{1-5y}{2y-5x}$$

16. Find the degree of homogeneity of the function $f(x, y) = \sqrt{x^4 + 3y^4}$.

- A. 0
- B. 1
- C. 2
- D. 4

$$t^4$$

Use the following information to answer questions 17 to 19.

The Jacobian of x and y with respect to u and v is $\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} 4 & 7 \\ 9 & 2 \end{vmatrix}$

$$\begin{matrix} x_u & x_v \\ y_u & y_v \end{matrix}$$

17. Find $\frac{\partial y}{\partial v}$.

- A. 2
- B. 4
- C. 7
- D. 9

18. Find $\frac{\partial x}{\partial u}$.

- A. 2
- B. 4
- C. 7
- D. 9

19. Find $\frac{\partial x}{\partial v} \frac{\partial y}{\partial u}$.

- A. 63
- B. 36
- C. 14
- D. 8

20. Find $f_y(x, y)$ if $f(x, y) = \ln(x^2 y)$.

- A. x
- B. $1/y$
- C. $1/x$
- D. $1/(xy)$

~~$\frac{1}{x^2 y}$~~
 $\frac{1}{x^2 y} \cdot x^2 = \frac{1}{y}$