

OCTOBER 2023
MAT 301SW
ADVANCED CALCULUS I
1 HOUR 20 MINUTES

Candidate's Index Number

1E/MAT/ST/22/0076

Signature:

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
SCHOOL OF EDUCATIONAL DEVELOPMENT AND OUTREACH
INSTITUTE OF EDUCATION

FIVE-SEMESTER BACHELOR OF EDUCATION (SANDWICH) PROGRAMME
LEVEL 350, END-OF-SECOND SEMESTER EXAMINATION, OCTOBER 2023

5TH OCTOBER 2023

ADVANCED CALCULUS I

9:40 AM - 11:00 AM

SECTION B
[40 MARKS]

Answer any TWO questions from this Section.
Please, note that if you answer more than two questions, only the first two will be marked.

1.

a. Evaluate $\lim_{(x,y) \rightarrow (0,0)} \frac{xy \cos y}{3x^2 + y^2}$.

[10 marks]

b. Show that the function $f(x,y) = \begin{cases} \frac{3x^2y}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$ is continuous at $(0,0)$.

[10 marks]

2.

a. If $f(x,y) = x \ln(x^2 + y^2)$, find $f_x(x,y)$ and $f_y(x,y)$.

[10 marks]

b. Evaluate the double integral $\iint_R \cos(x + 2y) dA$, where
 $R = \left\{ (x,y) : 0 \leq x \leq \pi, 0 \leq y \leq \frac{\pi}{2} \right\}$

[10 marks]

3.

a. Let $E = \{(r, \theta, z) : 0 \leq \theta \leq 2\pi, 0 \leq r \leq 1, 1 - r^2 \leq z \leq 4\}$ and $f(x, y, z) = \sqrt{x^2 + y^2}$.

i. Express $\iiint_E f(x, y, z) dV$ in cylindrical coordinates. [6 marks]

ii. Evaluate $\iiint_E f(x, y, z) dV$. [5 marks]

b. If $f(x, y, z) = 1$, evaluate $\iiint_E f(x, y, z) dV$ where E is a spherical wedge is given by

$E = \{(\rho, \theta, \phi) : 0 \leq \theta \leq 2\pi, 0 \leq \phi \leq \pi/4, 0 \leq \rho \leq \cos \phi\}$. [9 marks]

4. Let $f(x, y) = x^2 - 2xy + 2y$ be defined on the rectangle $D = \{(x, y) : 0 \leq x \leq 3, 0 \leq y \leq 2\}$.

a. Find the critical points of $f(x, y)$. [4 marks]

b. Find the extreme values of f on the boundary of D . [13 marks]

c. Find the absolute maximum and absolute minimum values of $f(x, y)$ on D . [3 marks]