

JANUARY 2023
MAT 301SW
ADVANCED CALCULUS I
1 HOUR 20 MINUTES

Candidate's Index Number

16/MAT/KM/21/0107

Signature: *[Signature]*

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
SCHOOL OF EDUCATIONAL DEVELOPMENT AND OUTREACH
INSTITUTE OF EDUCATION

FIVE-SEMESTER BACHELOR OF EDUCATION (SANDWICH) PROGRAMME
LEVEL 400, END-OF-FIRST SEMESTER EXAMINATION, JANUARY 2023

8TH JANUARY 2023

ADVANCED CALCULUS I

4:40 PM - 6:00 PM

SECTION B
[40 MARKS]

Answer any TWO questions from this section.

1. a. Evaluate $\lim_{(x, y) \rightarrow (0, 0)} \frac{xy^2}{x^2 + y^4}$. [12 marks]

b. Determine if the function

$$f(x, y) = \begin{cases} \frac{x^4}{x(x^2 + y^2)}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

is continuous at $(0, 0)$. [8 marks]

2. a. Given that $R = \ln(u^2 + v^2 + w^2)$, where $u = x + 2y$,
 $v = 2x - y$, and $w = 2xy$, use the Chain Rule to find [4 marks]

(i) $\frac{\partial R}{\partial x}$ [4 marks]

(ii) $\frac{\partial R}{\partial y}$

b. Evaluate $\iint_R (3x + 4y^2) dA$, where
 $R = \{(r, \theta) : 1 \leq r \leq 2, 0 \leq \theta \leq \pi\}$. [12 marks]

3. a. Use polar coordinates to evaluate

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2 + y^2} dy dx.$$

[12 marks]

b. Evaluate the triple integral $\iiint_B xyz^2 dV$, where B is the rectangular box given by $B = \{(x, y, z) : 0 \leq x \leq 1, -1 \leq y \leq 2, 0 \leq z \leq 3\}$.

[8 marks]

4. Use the change of variables $x = \frac{1}{2}(u+v)$ and $y = \frac{1}{2}(v-u)$ to evaluate the integral $\iint_R \frac{e^{(x-y)}}{x+y} dA$ where R is the rectangle bounded by the lines $y = x, y = x+5, y = 2-x$ and $y = 4-x$.

[20 marks]