DECEMBER 2021
MAT 301SW
ADVANCED CALCULUS I
2 HOURS

Candidate's Index Number
Signature:

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
SCHOOL OF EDUCATIONAL DEVELOPMENT AND OUTREACH
INSTITUTE OF EDUCATION

FIVE-SEMESTER BACHELOR OF EDUCATION (SANDWICH) PROGRAMME
LEVEL 350, END-OF-SECOND SEMESTER EXAMINATION, DECEMBER, 2021

DECEMBER 24, 2021 **ADVANCED CALCULUS I** **2:00 PM - 2:40 PM**

This paper consists of two sections, A and B. Answer ALL the questions in Section A and TWO questions from Section B. Section A will be collected after the first 40 minutes.

SECTION A

Answer all the questions in this Section.

For items 1 to 20, each stem is followed by four options lettered A - D. Read each statement carefully and circle the letter that bears the correct or best option.

1. Find the domain of the function $f(x,y) = y\ln(x-1)$.

   A. $\{(x,y): x > 1\}$
   B. $\{(x,y): x < 1\}$
   C. $\{(x,y): x \geq 1\}$
   D. $\{(x,y): x \leq 1\}$

2. Evaluate $\lim_{(x,y)\to(0,0)} \frac{xy}{2x^2+y^2}$ along the path $y = x$.

   A. Does not exist
   B. 3
   C. $\frac{1}{2}$
   D. $\frac{1}{3}$

3. Determine the set of points at which the function $f(x,y) = \frac{x-y}{2-x^2-y^2}$ is continuous?

   A. $\{(x,y) \in \mathbb{R}^2: (x,y) \neq (-1,-1)\}$
   B. $\{(x,y) \in \mathbb{R}^2: (x,y) \neq (1,1)\}$
   C. $\{(x,y) \in \mathbb{R}^2: (x,y) \neq (\pm1,\pm1)\}$
   D. $\{(x,y) \in \mathbb{R}^2 : (x,y) \neq (1,-1)\}$

4. Suppose that $f(2,1) = 4$. If $f(x,y)$ is continuous, find $\lim_{(x,y)\to(2,1)} f(x,y)$.

A. Does not exist
B. 2
C. 3
D. 4

5. If $f(x,y) = -\cos(2xy)$, find $f_x(x,y)$.

A. $2x\sin(2xy)$
B. $2y\sin(2xy)$
C. $-2y\sin(2xy)$
D. $-2x\sin(2xy)$

6. If $f(x,y) = e^{xy}\ln(4y)$, find f_x.

A. $xe^{xy}\ln(4y) + y^{-1}$
B. $ye^{xy}\ln(4y) + y^{-1}$
C. $ye^{xy}\ln(4y)$
D. y^{-1}

7. Find the second partial derivative f_{xy} of $f(x,y) = x^6 - y^3x$.

A. $-3y^2$
B. $6x^5 - y^3$
C. $6x^5 - 6xy$
D. $30x^4 - 3y^2$

8. If $z = f(x,y) = ye^{x+6}$, find the differential dz.

A. $ydx + e^{x+6}dy$
B. $xe^{x+6}dx + e^{x+6}dy$
C. $ye^{x+6}dx + e^{x+6}dy$
D. $e^{x+6}dx + ye^{x+6}dy$

9. Find the chain rule for $\frac{\partial z}{\partial s}$ if $z = f(x,y)$ with $x = g(s,t)$ and $y = h(s,t)$.

A. $\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{ds}{dy} + \frac{\partial z}{\partial y}\frac{ds}{dx}$
B. $\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$
C. $\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$
D. $\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{ds}{dt} + \frac{\partial z}{\partial y}\frac{dt}{ds}$

10. Find $\frac{dy}{dx}$ if $x^2 + y^2 = x$.

A. $-\frac{2x-1}{2y}$

B. $-\frac{2y}{2x-1}$

C. $-\frac{x}{y}$

D. $-\frac{2x+2y}{2x-1}$

11. Evaluate the double integral $\iint_R y dA$, where $R = \{(x,y): 0 \le x \le 2, 1 \le y \le 2\}$.

A. 1
B. 2
C. 3
D. 4

12. Evaluate $\iint_D 2dA$, where $D = \{(x,y): 0 \le x \le 1, x^2 \le y \le 1 + x^2\}$.

A. 0
B. 2
C. 3
D. 4

13. Evaluate $\iint_D 5dA$, where $D = \{(x,y): 0 \le y \le 1,\ 0 \le x \le y^4\}$.

A. 5
B. 3
C. 2
D. 1

14. If $f(x,y) = 2$, evaluate $\iint_R f(x,y)\, dA$, where $R = \{(r,\theta) : 1 \le r \le 2, 0 \le \theta \le 2\pi\}$.

A. π
B. 3π
C. 6π
D. 10π

15. Express $\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dy\, dx$ in polar coordinates.

A. $\int_0^{\pi/2} \int_0^{2\cos\theta} r^2\, dr\, d\theta$
B. $\int_0^{\pi/2} \int_0^{\cos\theta} r\, dr\, d\theta$
C. $\int_0^{\pi} \int_0^{2} r^3\, dr\, d\theta$
D. $\int_0^{\pi} \int_0^{2\cos\theta} r^2\, dr\, d\theta$

16. Express $\iiint_E z^2 dV$, where $E = \{(x,y,z): 0 \le x \le 2, 0 \le y \le x^2, 0 \le z \le x^2 + y^2\}$ in a form that can be evaluated.

A. $\int_0^{x^2+y^2} \int_0^{x^2} \int_0^2 z^2 dx\, dy\, dz$
B. $\int_0^{x^2} \int_0^2 \int_0^{x^2+y^2} z^2 dz\, dx\, dy$
C. $\int_0^2 \int_0^{x^2+y^2} \int_0^{x^2} z^2 dy\, dz\, dx$
D. $\int_0^2 \int_0^{x^2} \int_0^{x^2+y^2} z^2 dz\, dy\, dx$

17. Let $E = \{(r,\theta,z): 0 \le \theta \le \pi, 0 \le r \le 2, 1 \le z \le 2 + r^2\}$ and $f(x,y,z) = 1$. Express $\iiint_E f(x,y,z) dV$ in cylindrical coordinates.

A. $\int_0^{\pi} \int_0^2 \int_1^{2+r^2} r\, dz\ dr\ d\theta$
B. $\int_0^{\pi} \int_0^2 \int_1^{2+r^2} 1\, dz\ dr\ d\theta$
C. $\int_0^{\pi} \int_0^2 \int_1^{2+r^2} r^2\, dz\ dr\ d\theta$
D. $\int_0^{\pi} \int_0^2 \int_1^{2+r^2} r\, dz\ d\theta\ dr$

18. If E is a spherical wedge given by $E = \{(\rho,\theta,\phi): 0 \le \rho \le 3, 0 \le \theta \le 3\pi, 0 \le \phi \le 2\pi\}$, express $\iiint_E (x^2 + y^2 + z^2) dV$ in spherical coordinates.

A. $\int_0^{2\pi} \int_0^{3\pi} \int_0^3 \rho^2 \sin\phi\ d\rho\, d\theta\, d\phi$
B. $\int_0^{2\pi} \int_0^{3\pi} \int_0^3 \rho^4 \sin\phi\ d\rho\, d\theta\, d\phi$
C. $\int_0^{2\pi} \int_0^{3\pi} \int_0^3 \rho \sin\phi\ d\rho\, d\theta\, d\phi$
D. $\int_0^{2\pi} \int_0^{3\pi} \int_0^3 \sin\phi\ d\rho\, d\theta\, d\phi$

19. Let $f(x,y) = 4x^2 + 4y^2 - 24x - 16y + 7$. Find the critical point of f.
A. $(-2,3)$
B. $(2,-3)$
C. $(1,1)$
D. $(3,2)$

20. Suppose that f is continuous on a disk with center (a,b), such that $f_x(a,b) = 0$ and $f_y(a,b) = 0$. If $D(a,b) > 0$ and $f_{xx}(a,b) > 0$, then which of the following is true about $f(a,b)$?
A. local maximum.
B. local minimum.
C. saddle.
D. absolute maximum.

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SECTION B

Answer any **TWO** questions from this section.

1.

a. Evaluate $\lim\limits_{(x,y)\to(0,0)} \frac{xy\cos y}{3x^2+y^2}$. [10 marks]

b. Show that the function

$$f(x,y) = \begin{cases} \dfrac{3x^2y}{x^2+y^2}, & if \ (x,y) \neq (0,0) \\ 0, & if \ (x,y) = (0,0) \end{cases}$$

is continuous at $(0,0)$. [10 marks]

2.

a. If $f(x,y) = x\ln(x^2+y^2)$, find $f_x(x,y)$ and $f_y(x,y)$. [10 marks]

b. Evaluate the double integral $\iint_R \cos(x+2y)\,dA$, where
$R = \left\{(x,y): 0 \le x \le \pi, 0 \le y \le \frac{\pi}{2}\right\}$ [10 marks]

3.

a. Compute the second order Taylor formula for $f(x,y) = e^x \cos y$ about $(x_0, y_0) = (0,0)$. [10 marks]

b. If $f(x,y,z) = 1$, evaluate $\iiint_E f(x,y,z)dV$, where $E = \{(x,y,z): -1 \leq x \leq 1, 3x^2 \leq z \leq 4 - x^2, 0 \leq y \leq 6 - z\}$. [10 marks]

4.

a. Find the local maximum and minimum values and saddle points of $f(x,y) = x^4 + y^4 - 4xy + 1$.

[12 marks]

b. Evaluate $\iiint_E e^{(x^2+y^2+z^2)^{3/2}} dV$ where E is a spherical wedge is given by

$E = \{(\rho, \theta, \phi) : 0 \leq \rho \leq 1, 0 \leq \theta \leq 2\pi, 0 \leq \phi \leq \pi\}$. [8 marks]