

AUGUST 2020
MAT 301SW
ADVANCED CALCULUS I
1 HOUR 20 MINUTES

Candidate's Index Number

Signature:

UNIVERSITY OF CAPE COAST
COLLEGE OF EDUCATION STUDIES
SCHOOL OF EDUCATIONAL DEVELOPMENT AND OUTREACH
INSTITUTE OF EDUCATION

FIVE-SEMESTER BACHELOR OF EDUCATION (SANDWICH) PROGRAMME
SECOND YEAR, SECOND SEMESTER EXAMINATION, AUGUST 2020

AUGUST 29, 2020

ADVANCED CALCULUS I

2:40 PM - 4:00 PM

SECTION B

Answer any TWO questions from this section.

1. a. Evaluate $\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2+y^4}$. [12 marks]

b. Determine if the function

$$f(x, y) = \begin{cases} \frac{x^4}{x(x^2+y^2)}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$
 is continuous at $(0, 0)$. [8 marks]

2. a. Given that $R = \ln(u^2 + v^2 + w^2)$, where $u = x + 2y$, $v = 2x - y$, and $w = 2xy$, use the Chain Rule to find:

i. $\frac{\partial R}{\partial x}$ [4 marks]

ii. $\frac{\partial R}{\partial y}$ [4 marks]

b. Evaluate $\iint_R (3x + 4y^2) dA$, where $R = \{(r, \theta) : 1 \leq r \leq 2, 0 \leq \theta \leq \pi\}$. [12 marks]

3. a. Use polar coordinates to evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2 + y^2} dy dx$. [12 marks]

b. Evaluate the triple integral $\iiint_B xyz^2 dV$, where B is the rectangular box given by $B = \{(x, y, z) : 0 \leq x \leq 1, -1 \leq y \leq 2, 0 \leq z \leq 3\}$. [8 marks]

4. Use the change of variables $x = \frac{1}{2}(u + v)$ and $y = \frac{1}{2}(v - u)$ to evaluate the integral $\iint_R \frac{e^{(v-y)}}{x+y} dA$ where R is the rectangle bounded by the lines $y = x$, $y = x + 5$, $y = 2 - x$ and $y = 4 - x$. [20 marks]